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TWO-FLUID MODEL OF SUPERCONDUCTIVITY

John Bardeen
University of Illinois,
Urbana, Illinois
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Analogies between superconductivity and superfluidity in liquid helium have been noted by many people.[1,2] We wish to point out here that Landau's interpretation[3] of the two-fluid model of liquid helium in terms of motion of elementary excitations can be extended to provide a basis for a two-fluid model for superconductivity. The quasi-particle excitations of the Bardeen, Cooper, Schrieffer[4] (BCS) theory of superconductivity obey Fermi rather than Bose statistics, but the phenomenological equations of the two-fluid model are formally identical. The main assumption is that interactions involve only relative momenta, and so are unchanged by a uniform displacement of the whole system in momentum space. While this is strictly true for helium, it is only an approximation for metals because of the periodic field of the lattice. The physical content is, of course, quite different for the two cases. The connecting link is the common momentum in the ground state of the pairs in the BCS theory and of particles in the Einstein-Bose condensation of helium.

In superconductors there is a coherence distance of the order of 10^{-4} cm, and it is only when changes in motion occur slowly over distances of this order that point relations can be used. We shall be concerned here only with this limit, where the Pippard nonlocal relation between current and field goes over into the usual London equations[1]:

$$-c \operatorname{curl} \Lambda_T \vec{j}_S = \vec{H}; \quad \partial(\Lambda_T \vec{j}_S)/\partial t = \vec{E}. \qquad (1)$$

Here Λ_T is the London parameter, which in a free-electron approximation is equal to m/ne^2 at $T=0$ and increases to ∞ as $T \to T_c$. One may write $j_S = -e\rho_S v_S/m$, where ρ_S is the density and v_S the velocity of the superfluid component. At $T=0$, $\rho_S = \rho = nm$, and at some finite temperature,

$$\rho_S/\rho = 1 - \rho_n/\rho = \Lambda/\Lambda_T. \qquad (2)$$

The acceleration equation then becomes $\dot{\vec{v}}_S = -e\vec{E}/m$ at any temperature. An expression for (2) was derived by BCS by treating the external field as a perturbation.

An identical expression for ρ_S may be obtained by following Landau's derivation of the two-fluid model. We shall use a free-electron model, although the analysis can be extended readily to a more general Bloch model for the normal metal. The quasi-particle excitations have an energy $E(p) = (\epsilon^2 + \epsilon_0^2)^{1/2}$, where $\epsilon = (p^2/2m) - E_F$ is the Bloch energy measured relative to the Fermi surface and ϵ_0 is the temperature-dependent energy gap parameter. First consider the superfluid at rest (corresponding to a zero-momentum pairing condition) and suppose that the excitations (normal component) have a net momentum $\vec{J}_n$. Their distribution function, $f(\vec{p})$, is chosen to minimize the free energy, F, subject to the subsidiary condition

$$\vec{J}_n = \sum \vec{p} f(\vec{p}), \qquad (3)$$

Note that only "single" excitations in the BCS sense contribute to (3). If $\vec{v}$ is the Lagrange multiplier for $\vec{J}_n$,

$$\delta F - \vec{v} \cdot \delta \vec{J}_n = 0, \qquad (4)$$

giving, with a slight generalization of the procedure of BCS,

$$f(\vec{p}) = \frac{1}{1 + \exp\{[E(p) - \vec{v}\cdot\vec{p}]/kT\}}. \qquad (5)$$

For small $\vec{v}$, $\vec{J}_n$ is porportional to $\vec{v}$, and the

coefficient of proportionality is defined as the normal density, ρ_n. This gives

$$\rho_n = \lim_{\vec{v}\to 0} (\vec{J}_n/\vec{v}) = -\frac{4\pi}{3h^3}\int_0^\infty p^4 \frac{d}{dE}\left(\frac{1}{1+e^{E(p)/kT}}\right) dp, \quad (6)$$

which is identical with Landau's expression, except for use of the Fermi in place of the Einstein-Bose distribution function. It also agrees with that derived from (2) with the BCS value for Λ/Λ_T.

If the whole system is displaced in momentum space and moves with a velocity $\vec{v}_s$, the pairs have a common velocity $\vec{v}_s$. Defining $\vec{v}_n = \vec{v} + \vec{v}_s$ and $\rho_s = \rho - \rho_n$, it follows quite generally that for small v,

$$\vec{J} = \rho\vec{v}_s + \rho_n\vec{v} = \rho_s\vec{v}_s + \rho_n\vec{v}_n, \quad (7)$$

and the associated increase in free energy is

$$\Delta F = \tfrac{1}{2}\rho_n\vec{v}_n^{\,2} + \tfrac{1}{2}\rho_s\vec{v}_s^{\,2}. \quad (8)$$

The other equations of the two-fluid model of liquid He also follow. In particular, it is possible to construct states, in the way Feynman[5] has done for He, for which $\vec{v}_s$ is a slowly varying function of position. As for He, it is required that curl $\vec{v}_s = 0$.

The two-fluid model provides a basis for a discussion of persistent currents, particularly if we neglect magnetic effects. If there is a current-carrying state with $\vec{v}_s = 0$, scattering of individual particles can reduce $\vec{v}_n$ to zero, but such scattering will not change $\vec{v}_s$. Only a force which acts on all or a large fraction of the electrons can do so. A more complete discussion of persistent currents would require taking the Meissner effect into account and use of nonlocal relations.

Because of the very high attenuation of the normal component from interaction with the crystal lattice, phenomena such as second sound would be difficult to observe in superconductors.[2] It is possible that the peculiar results reported recently by Spiewak[6] on magnetic field dependence of high-frequency penetration may be associated with a counter flow of normal and superfluid components perpendicular to the surface in the penetration region. The wavelength of second sound in tin at the frequency she used (10^9 cps) is expected to be of the order of the penetration depth. However, use of local relations outlined above is questionable for this problem because, except very near T_c, the coherence distance is larger than the penetration depth.

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GENERATION OF PRISMATIC DISLOCATION LOOPS IN SILICON CRYSTALS

William C. Dash
General Electric Research Laboratory,
Schenectady, New York
(Received November 5, 1958)

The occurrence of helical dislocations in silicon was recently reported by the author.[1] They are formed when gold and copper are diffused simultaneously into the crystals at 1200°C, followed by quenching as in the method used for copper decoration.[2] Diffusion of gold alone forms helices which can be detected by etching. The detailed mechanism whereby these dislocations are formed has not yet been fully established. Similar helical dislocations have been observed in as-grown germanium crystals by Tweet.[3] In this case there is little doubt that climb induced by the condensation of vacancies is responsible for the phenomenon. For the purposes of this discussion it will be assumed that this process is also involved in the formation of helical dislocations in silicon crystals.

Helical dislocations have previously been observed in calcium fluoride crystals by Bontinck and Dekeyser.[4] It has been proposed that these dislocations result from climb processes in which vacancies are generated.[4-7]

Amelinckx et al.[6] have suggested that prismatic dislocation loops found in association with helices are formed by the cancellation of the screw component in the region of the loops by an interacting dislocation. Generation of similar loops by a mechanism involving only a single helical dislocation has been observed in silicon. Figure 1 shows a typical system which allows certain qualitative deductions about the generation of the dislocations to be made. The following mechanism is suggested:

At elevated temperature, there exists initially

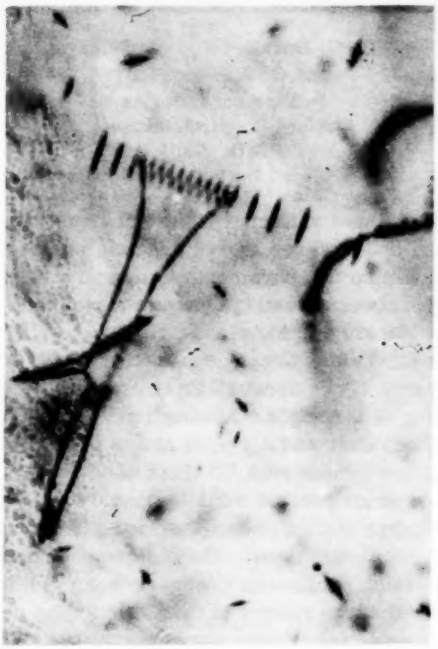

FIG. 1. Generation of prismatic loops from a helical dislocation. The dislocations have been decorated with copper and photographed with infrared sensitive plates. Separation of turns of the helix is about 10 microns.

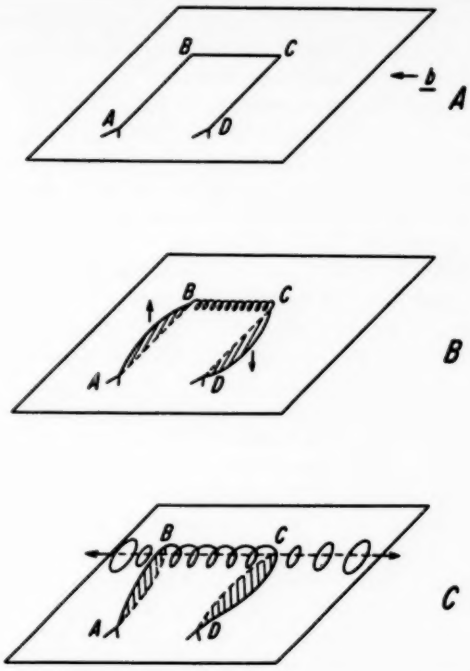

FIG. 2. Suggested mechanism for generation of prismatic loops. (a) Initial glissile dislocation loop *ABCD* with pinning points *A* and *D*. (b) Climb in the early stages of cooling. The dotted lines are the projections of *AB* and *CD* on the glide plane. *AB* and *CD* have moved perpendicular to the glide plane by acquiring a total number of vacancies equivalent to single planes of atoms indicated by the shaded areas. The screw portion *BC* has formed a helix with relatively small diameter. (c) After additional condensation of vacancies, the ratio between spacing of turns to diameter increases and causes mutual repulsion. This forces *AB* and *CD* apart. Some of the turns of the helix intersect *AB* and *CD* to form prismatic loops. The loops are repelled away from the helix and from each other into regions of higher vacancy concentration where they increase in diameter.

a glissile dislocation with pure edges *AB* and *CD*, each pinned at one end and joined together at the other end through screw *BC* [Fig. 2 (a)]. When the crystal is cooled, a supersaturation of vacancies appears. The dislocation loop climbs and thus acts as a sink for the excess vacancies. Initially, a uniform flux of vacancies converges upon all parts of the dislocation, driving *AB* and *CD* in opposite directions perpendicular to the glide plane and converting *BC* into a helix of many turns, as shown in Fig. 2 (b). Other authors[6,8] have considered the formation of a helical dislocation by climb. This Letter is not concerned with the mechanism of formation of a helix but rather with the modification of one which is already formed as vacancy condensation continues.

The entire configuration in Fig. 2 (b) can glide parallel to the Burgers vector b even though it is no longer confined to a single plane. The chemical stress produces a mechanical stress in the form of repulsion between adjacent turns of the helix. The repulsion can be relieved by glide of the turns over the surface of a cylinder coaxial with the helix. The helix expands and forces *AB* and *CD* apart. The angle between *AB* and *CD* and the axis of the helix decreases steadily and eventually successive turns of the helix intersect *AB* and *CD* and form closed loops. The closed loops are repelled from the helix and from each other and can glide on a cylinder concentric with the helix. Their diameter will increase by further climb as they glide into regions of relatively higher excess vacancy concentration.

With a sufficient initial supersaturation in vacancies, a large number of coaxial prismatic loops can be derived from this source.

It is interesting to observe in Fig. 1 that the closed loops are more widely spaced than the turns of the helix. This would be anticipated because forces between turns of the helix are balanced by the oppositely directed forces acting on AB and CD.

It follows from Fig. 1 and from the above analysis that there is mechanical equilibrium between the various parts of the dislocation loop. However, there is no evidence of a balance between chemical and mechanical stresses. If such a balance did exist, the relatively low line tension of edge dislocations with large radii of curvature would enable them to acquire vacancies at the expense of helices and small prismatic loops. This is not observed. The total climb of the edges is not significantly different from that represented by the sums of the areas of the prismatic loops and of the projected areas of the turns of the helix. This indicates that steady-state conditions do not prevail and therefore Weertman's[8] prediction of the final form of the helices does not apply. Instead, a dynamical theory is required to explain the observations.

The mechanism described appears to account for many of the observed sequences of coaxial prismatic loops; a series of over a hundred loops is not unusual. In other cases large numbers of coaxial prismatic loops are found to be completely isolated from other dislocations. The generating mechanism in such cases is not known.

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SHOCK-WAVE COMPRESSION OF ALUMINUM

D. G. Doran,* G. R. Fowles, and G. A. Peterson†
Poulter Laboratories,
Stanford Research Institute,
Menlo Park, California
(Received November 6, 1958)

Use of a new reflected-light technique with oblique shock waves allows dynamic pressure-compression measurements to be made in the region between existing dynamic measurements (50 to 500 kb) and the elastic regime. Previous methods[1,2] using argon gaps were restricted to pressures above about 50 kb in order to produce flashing of the argon. Although pin techniques have been successfully used at low pressures with plane geometries,[3-5] their use with the oblique shock method would seriously restrict the number of data points obtained. (When used with optical techniques, the oblique shock method provides continuous pressure-compression data over large ranges of pressure.) The new technique is free of the above restrictions and thus provides a method for examining the transition between elastic and "hydrodynamic" behavior. Further, considerably greater overlap with Bridgman's[6,7] static data than now exists can be obtained. This possibility is of fundamental significance since several experimentally unevaluated assumptions are necessary to the interpretation of shock-wave results. The feasibility of the method has been demonstrated for aluminum.

Shock and free-surface velocity measurements were made along the center line of the face of a wedge-shaped specimen backed by a line-initiated slab of high explosive as shown in Fig. 1. Rotating-mirror streak-camera[8] observations of a light beam reflected from the aluminum-coated Mylar foil and from the clear Mylar film give the arrival times T_1, of the shock, and T_2, of the free-surface, respectively, at points down the wedge face. Air shock complications were removed by evacuating the foil-film system. (It is estimated that maximum displacement of the outer foil due to reflection of the remaining air shock amounts to less than 5 microns.)

The detonation velocity U_d was determined by measuring the time of closure by the detonation ionization front of silver foil switches placed along the explosive slab. The shock velocity at a distance z down the wedge face is given by $U_S = U_d \sin\alpha$, where $\alpha = \operatorname{arccot}(\cot A + C\,U_d$

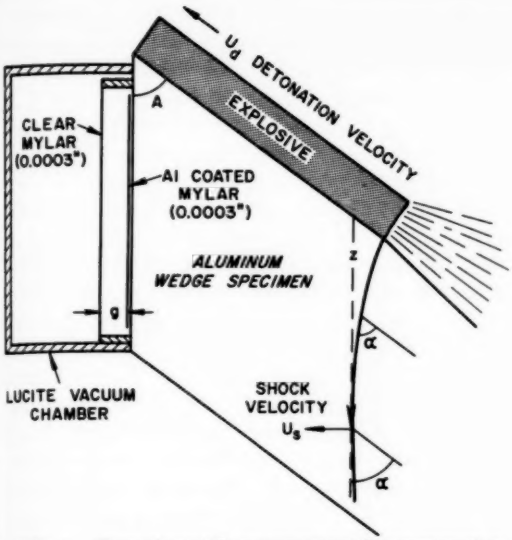

FIG. 1. Experimental arrangement for measuring shock and free-surface velocities.

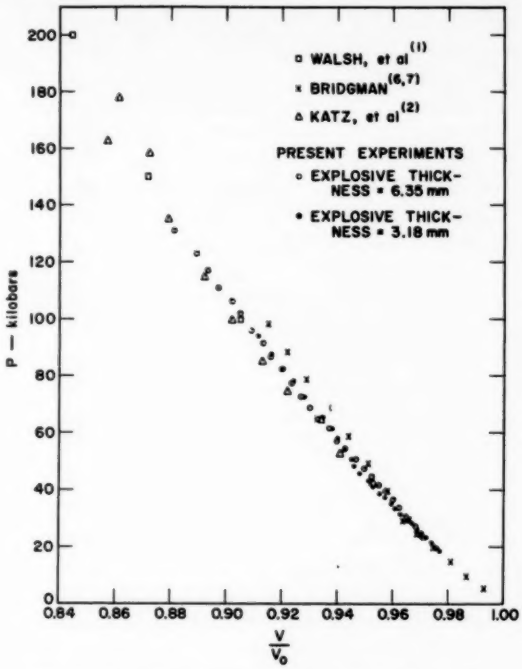

FIG. 2. Pressure-compression points for aluminum.

$\times\cot\gamma\csc A$) is the shock angle, C is a constant of the streak-camera optical system, and γ is the angle of the film smear trace with respect to the time axis. The free-surface velocity is $U_{fs} = g/(T_2 - T_1)$ where $T_2 - T_1$ is the time required for the free-surface to traverse the distance g.

Assuming that (1) thermodynamic equilibrium is attained in times short compared with other times of interest ($< 10^{-7}$ sec), (2) the stress is isotropic, and (3) the free-surface velocity is twice the particle velocity, U_p, behind the shock,[1] corresponding pressure-compression points are found by the Rankine-Hugoniot relations $P = \rho_0 U_S U_p + P_0$ and $V/V_0 = (U_S - U_p)/U_S$ for the conservation of mass and momentum across a shock front. Here P_0, V_0, and ρ_0 refer to pressure, specific volume, and density ahead of the shock front, and P and V denote pressure and specific volume behind the shock front. The assumption of isotropy in stress will not be expected to hold at low pressures due to the finite yield strength of the material. This effect remains to be investigated.

The results shown in Fig. 2 agree well with other shock measurements in both plane[1] and oblique geometries[2] and Bridgman's static measurements[6,7] below about 50 kilobars.

Internal agreement was obtained with $g = 2.57$ mm and with $g = 1.30$ mm with explosive thicknesses of, respectively, 6.35 mm and 3.18 mm. The estimated error due to experimental uncertainties is 0.5% in V/V_0 for a given pressure. The observed experimental scatter is less than 0.3% in V/V_0 at a given pressure. A double arrival at the aluminized foil was observed in one shot. The first arrival was identified with the elastic precursor traveling at 6.45 mm/μsec.

Additional advantages to the reflected-light technique over argon gaps are (1) improved time resolution due to the sharpness of the trace, (2) elimination of the necessity of making shock and free-surface measurements at different points on the specimen face and, therefore, of consequent uncertainties due to curvature of the shock, and (3) reduction in minimum specimen size.

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*Now at Washington State College, Pullman, Washington.

†Now at Stanford University, Stanford, California.

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EXPERIMENTAL EVIDENCE FOR CARRIERS WITH NEGATIVE MASS

G. C. Dousmanis, R. C. Duncan, Jr., J. J. Thomas, and R. C. Williams
RCA Laboratories, Radio Corporation of America, Princeton, New Jersey
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Cyclotron resonance experiments, suggested earlier[1] for the detection of negative-mass carriers, have been carried out with Ge crystals at 4°K. Use of circularly polarized microwaves has revealed a new spectrum of heavy holes which, from the tests tried so far, may be assigned to carriers of negative mass. This spectrum is of the emission type: it appears as a "dip" in the background absorption, rather than as an absorption peak.

It has been pointed out[2] that, because of the re-entrant type of energy contour in Ge and Si, the transverse effective mass of heavy holes close to the [100] axis should be negative. Calculations have shown that the number of negative-mass carriers at equilibrium should be substantial and it was suggested that they be detected by cyclotron resonance.[1] This type of negative-mass effect is expected to persist at low energies and seemed more promising for experimental detection than the effect expected from carriers in the upper parts of simple band structures.[3,4]

The negative-mass holes would not yield a simple cyclotron line because of the large spread in mass (from about $-0.2\,m_0$ to $-\infty$ in Ge). Despite this, one calculates a single but broad and asymmetric line with the half-width on the high-field side larger than that on the lower. If one denotes by H_0 the field value for resonance of the positive-mass holes ($m^* = 0.28\,m_0$), the maximum of the negative-mass interaction should occur in the region of $-0.8H_0$ to $-2H_0$. The negative-mass interaction would produce a dip on the background absorption rather than an absorption peak since the negative-mass carriers are thrown into lower energies. It is, of course, obvious that no net emission will result if strict thermal equilibrium prevails. In the absence of any information the relaxation time for negative masses is assumed to be comparable to that for positive ones. Since the center of the negative-mass cone[1] offers a line of constant m^* [its value is predicted in the range of $(-0.2$ to $-0.5)\,m_0$], with H in the [100] direction, one may expect the observable m^* to represent roughly that of the cone axis.

The apparatus used in these experiments is similar to that of other workers,[5,6] except that no use is made of phase-sensitive detection. The magnetic field is applied along the [100] axis. In contrast to most earlier work, H is perpendicular to the plane of the samples (disks 0.5 cm in diameter and 0.03 cm thick). It is not clear whether this arrangement is preferable, but surface and plasma (depolarization) effects must be different in the two cases. The carriers are generated by illumination with a tungsten lamp. Illumination provides input energy to the specimen. Majority carriers are preferentially generated with infrared. Very few majority carriers are generated by impact ionization in the applied rf field. The microwave power is less than one milliwatt.

Data taken with a 10-ohm-cm p-type Ge sample that yields narrow spectra are shown in Fig. 1. On the side marked $+H$, spectra with positive q/m^* are detected. On the $-H$ side one expects

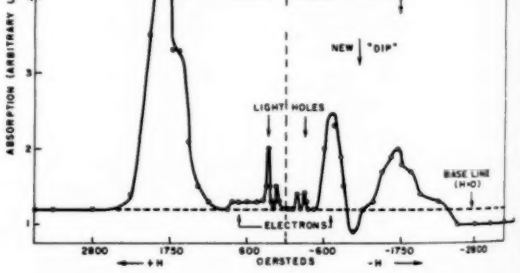

FIG. 1. Cyclotron resonance spectra of Ge (taken with circularly polarized waves) and the new structure ("dip") on the $-H$ side. A weak absorption peak is also seen near the light holes. H is oriented along the [100] axis. The dip here, seen also in the subsequent figures, is assigned to carriers with negative mass.

electron spectra and evidence for negative-mass heavy holes (q/m^* negative). Since the circular polarization is not complete, traces of spectra appear even on the "wrong" side of H. In Fig. 1 a new structure is seen where negative-mass holes are expected. The structure consists mainly of a "dip" at $H \sim -0.7H_0$. Since the dip occurs between the electrons and the heavy holes it can be easily distorted by the former or the latter depending on the electron concentration and the degree of polarization, respectively. To our knowledge there are no data in the literature for comparison with the $-H$ side or the low-field $+H$ side. In several cases a small peak appears at $\pm 1.9H_0$ which had been seen earlier.[7,8] A new absorption peak appears near the light holes and is more prominent on the $+H$ side. This peak is weak and its exact determination and interpretation will require further work.[9] Structure is also indicated on the low-field side of the heavy-hole line (Fig. 1). The data are taken with an undercoupled cavity. That the dip is not simply a continuation of the electron line is shown by the data of Fig. 2, taken with less rf power, much better polarization, and the cavity undercoupled but further from match. It is seen that the dip is separate from both the electrons and the residual heavy-hole resonance. The dip is strong enough to displace to the right the heavy holes on the $-H$ side. The apparent small shifts of the spectra from one run to another are mainly due to the manual setting of the coil current.

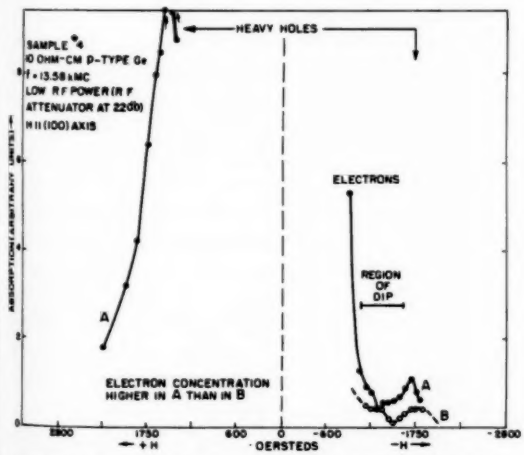

FIG. 2. Part of the spectra of Ge showing the region of the new dip. By comparison to Fig. 1, the microwave power here is lower, the polarization more complete, and the cavity coupling further away from match.

The dip may be interpreted as a new resonance for heavy holes. Referring to Fig. 1, reference 1, with H along the k_x axis one would expect two resonances from the heavy holes, corresponding respectively to m_1 (usual heavy-hole mass) and m_2 (points along k_x, the re-entrant contours). m_2 is negative and is associated with the dip on the $-H$ side. For single mass values, m_1 and m_2, one calculates, using the constants of earlier work,[5,6] for H along the [100] axis,

$$|m_1/m_2| \sim |A-B-(C^2/2B)|/[A-(B^2+C^2/6)^{1/2}]$$

$$\sim 0.6 \pm 0.2 \text{ from } A, B, C \text{ of reference 5}$$

$$\sim 1.4 \pm 0.4 \text{ from } A, B, C \text{ of reference 6.}$$

The experimental value, an average of several measurements, is 1.3 ± 0.3. The m_2 interaction is weak since under resonance E_{rf} may tend to change the carrier mass, more so than in the case of m_1.

Negative-mass spectra, by comparison to the absorption spectra, are possibly detected over a different range of the instrumental curve of reflected power vs rf conductivity. As one approaches negative-mass resonance the sample conductivity presumably tends to decrease rather than increase as with the absorption spectra of positive-mass carriers. Hence, depending on the linearity of the instrument, negative-mass effects may be somewhat different from that predicted from the usual spectra.

The shape and intensity of the emission-type spectrum are sensitive to microwave power. Figure 3 shows data taken with another sample

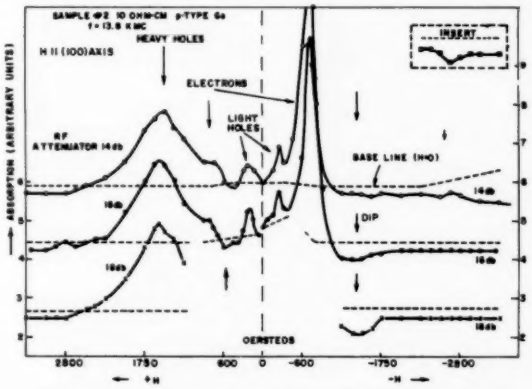

FIG. 3. The spectra of Ge taken with another 10-ohm-cm p-type sample. Note that as the microwave power is progressively decreased the new spectrum (dip) becomes pronounced. The insert shows a smaller dip observed at higher fields ($m^* \sim -0.48\, m_0$).

and with more electrons present. The polarization here is better than in Fig. 1. As the rf power is progressively decreased the "dip" becomes pronounced. No evidence of the heavy-hole line is seen on the $-H$ side, presumably as a result of the subtracting new signal. A smaller effect, similar to the main dip, appears on the $+H$ side (Fig. 3). The effect of the rf may be due partly to impact ionization, but a strong rf may also tend to depopulate the negative-mass cone.[1] This would correspond to power saturation in other types of emission spectra.

The new spectrum taken with a 30-ohm-cm sample is shown in Fig. 3, Part A. The higher resistivity does not necessarily imply better crystalline quality and narrower spectra. The "dip" is reduced in intensity if H is misoriented by about 15° from the [100] axis, as expected from the small angle of the negative-mass cone.[1] Such a run, where the new spectrum is not seen, is shown in Fig. 4, part B.

With H at values outside the new "dip," the conductivity σ_{rf}, as measured from the reflected rf power, increases monotonically with illumination I, starting with $I = 0$. When $-H$ was set at the position of the main dip, σ_{rf} did not change with I and in some cases it even decreased with increasing I. A similar, but less pronounced, tendency has been found with the dip following the light holes on the $+H$ side. This suggests the presence of a negative conductivity component that tends to cancel the increase in σ_{rf} expected from the increased numbers of the usual carriers. That the "dip" in Fig. 3 is lower than the high-field absorption limit (on either side of H) may again be indicative of negative conductivity. Further measurements at higher fields are required to establish whether the limits indicated by the data represent truly zero rf conductance.

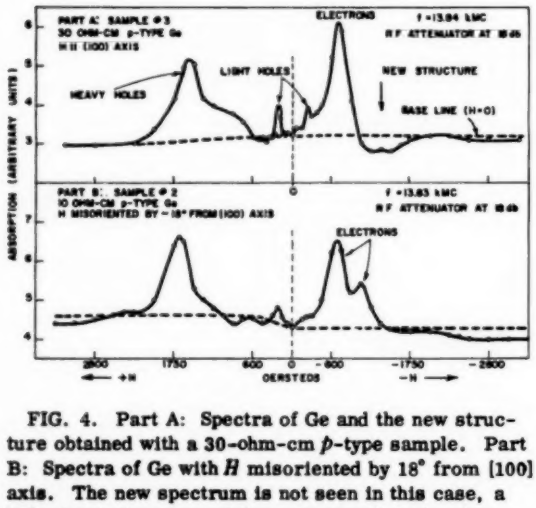

FIG. 4. Part A: Spectra of Ge and the new structure obtained with a 30-ohm-cm p-type sample. Part B: Spectra of Ge with H misoriented by 18° from [100] axis. The new spectrum is not seen in this case, a behavior to be expected from the small angle of the negative-mass cone.

The lack of correlation between electron intensity and the new "dip" makes it unlikely that this effect is due to electrons. In any case, the electron cyclotron spectra in either the ground or excited states should be of the absorption type. The rf power and the temperature are too high for occurrence of quantum effects.[8-10] Complications from magnetoresistance are unlikely because of the low carrier densities. The small dip on the $+H$ side may be assigned to the extreme low-field end of the negative-mass spectrum detected with small intensity on that side.

The main dip occurs at $m^* = -(0.22 \pm 0.06)m_0$. A smaller dip (see insert, Fig. 3) has also been observed at $m^* \sim -0.48m_0$.

In summary, a new resonance has been found for the heavy holes in Ge. The new heavy-hole effect[11] is more prominent on that side of H that is appropriate for detection of negative-mass holes. The direction of the effect (decreased absorption) and its spectral position, approximately, are as expected. The intensity of the effect varies with microwave power. All observations on intensity fall well within an order of magnitude of the earlier prediction,[1] but mainly on the upper magnitude side. The reentrant part of the contours should be given more accurately by detailed analysis of the present data. The sensitivity to rf power suggests that the dependence on temperature and relaxation time should be of much interest. The energy relations and carrier distribution under which this emission-type effect occurs have not as yet been evaluated in detail. The very magnitude of the new effects in Ge offers promise for further band structure studies in this as well as other crystals.

The authors are indebted to Professor Charles Kittel for a helpful discussion, and to many members of the RCA Laboratories for their cooperation.

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THEORY OF THE MEISSNER EFFECT

D. Pines* and J. R. Schrieffer†
Ecole d'Eté de Physique Théorique,
Les Houches (Haute Savoie), France
(Received November 7, 1958)

In order to obtain an explicitly gauge-invariant description of the Meissner effect in superconductors, Wentzel[1] has recently suggested an approach which differs in several essential respects from that developed by Bardeen, Cooper, and Schrieffer.[2] Wentzel finds in the long-wavelength limit a relation between the current density and the magnetic vector potential which differs from the London value given by the BCS theory. We wish to point out that this discrepancy is due to Wentzel's assumption that corrections to his approach possess a convergent expansion in powers of the phonon-electron coupling constant, g. It is our belief that this expansion does not exist.

Wentzel considers the Hamiltonian

$$H=H_0+H_g+H_A\equiv H'+H_A, \qquad (1)$$

where the three terms describe the free electrons and phonons, the electron-phonon interaction of strength g, and the coupling of the electrons to the vector potential, A. Coulomb interactions between electrons are neglected. By carrying out a perturbation theoretic canonical transformation based on the unperturbed Hamiltonian, H_0, Wentzel eliminates the coupling of the system to the vector potential through order g^2. Thus he finds

$$\tilde{H}=e^{-K}He^{K}=H_0+H_g+O(g^3A), \qquad (2)$$

where

$$K=K_A+K_{gA}+K_{ggA}, \qquad (3)$$

and the generators are defined by

$$[H_0,K_A]=-H_A, \qquad (4)$$

$$[H_0,K_{gA}]=-[H_g,K_A], \qquad (5)$$

$$[H_0,K_{ggA}]=-[H_g,K_{gA}]. \qquad (6)$$

To the same order in g, the transformed current density operator is given by

$$\tilde{j}_\mu(\vec{q})=j_\mu(\vec{q})+[j_\mu(\vec{q}),K], \qquad (7)$$

where $j_\mu(\vec{q})$ is the current density operator in the presence of the vector potential in the original representation. Wentzel then calculates the current density by taking the expectation value of (7) with respect to the Bogoliubov[3]-Valatin[4] representation of the BCS ground state appropriate to $H'\cong\tilde{H}$. Since H_g is not treated as a perturbation in this last step, a Meissner effect is obtained and it is found that

$$\lim_{q\to 0} J_\perp(\vec{q})=-\frac{\rho}{2}\frac{ne^2}{m}A_\perp(\vec{q}), \qquad (8)$$

where $\rho=g^2N(0)\simeq\frac{1}{4}$, $N(0)$ being the density of states in energy of electrons of one spin at the Fermi surface and $\perp$ denotes the transverse components.[5]

To understand the difficulties with this approach, we note that the exact expression for the current density including terms linear in A may be written as[6]

$$J_\mu(\vec{q})=(\Psi_0|j_\mu(\vec{q})|\Psi_0)+(\Psi_0|[j_\mu(\vec{q}),S_A](\Psi_0)\equiv\sum_\nu K_{\mu\nu}(\vec{q})A_\nu(\vec{q}), \qquad (9)$$

where

$$[H',S_A]=-H_A, \qquad (10)$$

and Ψ_0 is the ground-state eigenfunction of H':

$$H'\Psi_\alpha=W_\alpha\Psi_\alpha. \qquad (11)$$

If S_A were expanded in powers of g, one would obtain Wentzel's expression for the current den-

sity. The exact expression for S_A is

$$(\Psi_\beta|S_A|\Psi_\alpha) = -\frac{(\Psi_\beta|H_A|\Psi_\alpha)}{W_\beta - W_\alpha} \quad (W_\beta \neq W_\alpha). \quad (12)$$

As Schafroth[7] has shown, the Meissner effect is established by considering the form of $K_{\mu\nu}(\vec{q})$ in the limit $q^2 \to 0$. We argue that for a superconductor, the excitation energies that appear in (12) have a radically different behavior from the free-electron excitation energies in this limit. Thus, the free-electron energies, $\epsilon_{k+q} - \epsilon_k = \vec{q}\cdot(2\vec{k}+\vec{q})/2m$, vanish in the limit of small q^2 while the superconductor possesses a finite energy gap, $2\epsilon_0$, for the single-particle-like excitations involved.

Wentzel's procedure is equivalent to assuming that to lowest order the excitation energies which appear in (12) may be replaced by their free-electron counterparts and that higher-order corrections may be included by a systematic expansion in powers of g. It is clear, however, that such an expansion will not converge in the limit $q^2 \to 0$, since the expansion parameter is of the order of $(m\epsilon_0/\vec{q}\cdot\vec{k})$. The BCS approach corresponds to replacing H' by the reduced or quasi-particle Hamiltonian. Indeed, it is the fact that $\vec{q}\cdot\vec{k}/m\epsilon_0 \to 0$ in this limit which is responsible for the London value of $K_{\mu\nu}(\vec{q})$ given by the BCS theory. In a more complete treatment, the BCS kernel would be corrected for the change in density of states due to the phonon-electron interaction, as shown by Rickayzen.[8] This correction should vanish for weak coupling, leading back to the London value, while Wentzel's approach would give a kernel which becomes vanishingly small in this limit.

The BCS approach will lead to gauge-invariant results only if the approximate eigenstates of H' are sufficiently accurate to satisfy the longitudinal f-sum rule. At present the BCS-Bogoliubov-Valatin states are not that accurate. This apparent lack of gauge invariance is resolved when the Coulomb interactions between electrons are included, since the plasmons play a dominant role in satisfying the sum rule in the long-wavelength limit. The authors have given elsewhere[9] an extension of the BCS theory of the Meissner effect in an arbitrary gauge and have shown the BCS result to be essentially unaltered.

It is a pleasure to thank Dr. Cecile DeWitt and Dr. Philippe Nozieres for the gracious hospitality extended to us by the Ecole d'Eté de Physique Théorique. We also thank Dr. G. Wentzel for a preprint of his paper and for valuable discussions.

*National Science Foundation Senior Post-Doctoral Fellow, on leave of absence from Princeton University, 1957-58; present address: The Institute for Advanced Study, Princeton, New Jersey.

†National Science Foundation Post-Doctoral Fellow, on leave of absence from the Department of Physics and The Institute for the Study of Metals, University of Chicago, Chicago, Illinois, 1957-58.

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CYCLOTRON RESONANCE IN FLAMES*

Jürgen Schneider and F. W. Hofmann
Department of Physics, Duke University,
Durham, North Carolina
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Cyclotron resonance of electrons in a gas discharge has been detected by Ingram and Tapley.[1] We have observed this effect in low-pressure flames at a frequency near 24 kMc/sec. Starting from the classical equation of motion of an electron moving in a homogenous, constant magnetic field H and an alternating electric field one obtains, for the complex electric conductivity σ perpendicular to H,

$$\sigma = \frac{Ne^2}{m} \frac{\nu + i\omega}{(\nu + i\omega)^2 + \omega_c^2}, \quad (1)$$

where e and m are the electronic charge and mass, N is the number of free electrons per cc, ω the frequency of the applied electric field, ν the collision frequency of electrons with neutral molecules, and $\omega_c = eH/mc$ the cyclotron frequency. The same equation can be deduced from the Boltzmann transfer equation assuming ν is independent of the electron velocity. Cyclotron resonance will occur near $\omega_c = \omega$ if the electron is able to make at least one revolution in the magnetic field before colliding with another particle, i.e., for $\omega/\nu > 1$. In an acetylene-air flame we found at K-band frequencies and at atmospheric pressure no indication of a cyclotron resonance, whereas at 100 mm Hg a broad absorption appeared which became considerably sharper as the pressure was decreased. Figure 1 shows

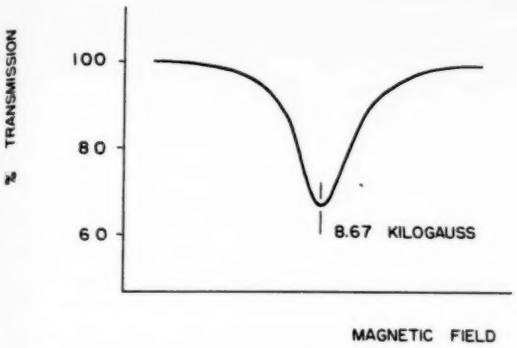

FIG. 1. Cyclotron resonance observed at 24.3 kMc/sec. The total field sweep is about 4800 gauss, the half-width 700 gauss.

a recorder trace of the cyclotron resonance line observed from an acetylene-oxygen flame at 7.5 mm Hg. The low-pressure vessel containing the burner was placed in an electromagnet between two waveguide horns, the one leading to the klystron, the other to the detector. By introducing into the flame fine sprays of alkali salt solutions from an atomizer, it was possible to vary the concentration of free electrons over a wide range. At low pressures where the line is sharp, the sensitivity of this method is sufficiently high to detect the free electrons which originate in the primary reaction zone of an acetylene-oxygen flame.

Equation (1) indicates that it is possible to determine both the electron concentration N and the collision frequency ν by this method. N is given by the integral

$$\int_0^\infty \sigma' dH = \frac{\pi}{2} ecN, \qquad (2)$$

where σ', the real part of the complex conductivity σ, is proportional to the measured absorption providing N is sufficiently small. The value of ν can be obtained from the shape of the line, e.g., from the half-width or from the ratio of the absorption at zero magnetic field and at the field for which maximum absorption occurs. It can be shown that Eq. (2) is also true for the case where the collision frequency ν is dependent on the electron velocity.

A full account of this work will be published elsewhere.

These investigations arose in connection with projects in microwave spectroscopy of Dr. Walter Gordy and flame spectroscopy of Dr. Hedwig Kohn.

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BOSE-EINSTEIN CONDENSATION AND THE λ-TRANSITION IN LIQUID HELIUM*

M. S. Green
Weizmann Institute of Science,
Rehovot, Israel†

(Received September 15, 1958)

Recently Yang[1] and Lee[2] have given a method of computing the grand partition function of an imperfect Bose gas valid for such low densities and temperatures that the mean distance between particles, $\rho^{-1/3}$, and the mean deBroglie wavelength, λ, are both long compared to the length scale of intermolecular forces, a, but for which the ratio $\rho^{1/3}\lambda$ may be of order of magnitude 1. The method cannot, however, be used directly in the neighborhood of the Bose-Einstein transition or when Bose-Einstein condensation is actually present. This letter reports a rearranged form of the Yang-Lee series which is valid below, at, and above the Bose-Einstein transition and compares theoretical conclusions about this transition to the experimental results of Fairbank, Buckingham, and Kellers[3] on the λ-transition in liquid helium.[4]

The transformed expression for the logarithm of the grand partition function is[5]

$$\log \Xi(z,\beta) = \sum_l \log \frac{1}{1 - z\exp[-\beta\epsilon(l)](1+\varphi_1(l))} - \sum_l n(l)\varphi_1(l) + \varphi_0, \qquad (1)$$

where l runs over possible values of the linear momentum, $\beta = 1/kT$, z is the absolute activity, and $\epsilon(l)$ is the kinetic energy;

$$n(l) = \frac{z\exp[-\beta\epsilon(l)]}{1 - z\exp[-\beta\epsilon(l)](1+\varphi_1(l))}, \qquad (2)$$

$$\varphi_0 = \sum_{\gamma=2}^{\infty} \frac{1}{\gamma!} \sum_{l_1 \ldots l_\gamma} W_\gamma(l_1 \ldots l_\gamma) n(l_1) \ldots n(l_\gamma), \qquad (3)$$

$$\varphi_1(l) = \sum_{\gamma=1}^{\infty} \frac{1}{\gamma!} \sum_{l_1 \ldots l_\gamma} W_{\gamma+1}(l, l_1 \ldots l_\gamma) n(l_1) \ldots (l_\gamma), \quad (4)$$

where the quantities W_γ are functions of the l's and β computable by the graph methods of Yang and Lee. Explicit formulas will be given in a more complete publication.

Formulas (1) to (4) are valid in both the condensed and uncondensed states, the condensed state being distinguished by the fact that $n(0)$ is of the order of magnitude of the total number of particles. It is convenient in the latter case to write Eq. (2) for $l = 0$ as

$$1 - z[1 + \varphi_1(0)] = z/n(0).$$

In the limit of a large system the right-hand side can be set equal to zero and we have

$$1 - z[1 + \varphi_1(0)] = 0. \quad (5)$$

In the uncondensed state Eq. (2) is an integral equation for determining the function $n(l)$. In the condensed state Eq. (5) provides the additional condition needed to determine $n(0)$.

Differentiation of Eq. (1) with respect to z and β yield the average number, N, and energy, E:

$$N = z \frac{\partial}{\partial z} \log \Xi = \sum_l n(l)[1 + \varphi_1(l)], \quad (6)$$

$$E = -\frac{\partial}{\partial \beta} \log \Xi = \sum_l n(l)[1 + \varphi_1(l)]\epsilon(l) - \sum_{\gamma=2}^{\infty} \frac{1}{\gamma!} \sum_{l_1 \ldots l_\gamma} \frac{\partial W_\gamma(l_1 \ldots l_\gamma)}{\partial \beta} n(l_1) \ldots n(l_\gamma). \quad (7)$$

In the condensed case we write for Eq. (6)

$$N = n(0)[1 + \varphi_1(0)] + \sum_{l \neq 0} n(l)[1 + \varphi_1(l)]. \quad (8)$$

It is not very difficult to understand how the Bose-Einstein transition in an imperfect gas can be higher than first order (N and E continuous). The activity of transition z_0 is defined as the root of Eq. (5) in the uncondensed state [$n(0) \equiv 0$]. Larger (smaller) z's correspond to the condensed state, Eq. (5) being maintained by making $n(0)$ a finite fraction of all molecules. This fraction is a function of z continuous at z_0. From Eqs. (6), (7), and (8), N and E are continuous at z.

It is more difficult to understand how the second derivatives $\partial E/\partial z$, $\partial E/\partial \beta$, $\partial N/\partial z$ can become infinite along the transition line in z, β space

$$z = z_0(\beta).$$

Indeed, the form for the grand partition function given above together with certain very reasonable assumptions seem to require finite values for the second derivatives along the transition line except perhaps for isolated points. This can be seen most easily for the derivative $z(\partial N/\partial z)$ which is essentially the isothermal compressibility. Differentiation of Eq. (6) yields

$$z \frac{\partial N}{\partial z} = \sum_l z \frac{\partial n(l)}{\partial z} [1 + \varphi_1(l) + \sum_{l'} \varphi_2(l, l') n(l')], \quad (9)$$

where

$$\varphi_2(l, l') = \sum_{\gamma=0}^{\infty} \frac{1}{\gamma!} \sum_{l_1 \ldots l_\gamma} W_{\gamma+2}(l, l', l_1 \ldots l_\gamma) \times n(l_1) \ldots n(l_\gamma). \quad (10)$$

We suppose that any infinity of $z\partial N/\partial z$ is associated with the divergence of the part of the sum on the right in Eq. (9) which comes from small momenta, since the vanishing of the denominator in Eq. (2) can be expected to cause anomalies only for low momenta.

We suppose, also, that the quantity in parentheses in Eq. (9) has a finite nonzero value for $l = 0$ at the transition. We investigate the sum

$$\sum_{|l| < \Delta} z \frac{\partial n(l)}{\partial z},$$

where Δ is a small momentum.

Differentiating Eq. (2) we have

$$z \frac{\partial n(l)}{\partial z} = n(l) + n^2(l)[1 + \varphi_1(l) + \sum_{l'} \varphi_2(l, l') z \frac{\partial n(l')}{\partial z}]. \quad (11)$$

Even though $n(l)$ is singular for $l = 0$ at the transition, the sum $\sum_l n(l)$, where $\sum_l$ implies summation over all l for which $|l| < \Delta$, must converge and approach 0 as $\Delta \to 0$, since N must be finite at the transition. $\sum_l n^2(l)$, however, may and, indeed, in the case of perfect gas does diverge. We suppose, first, that it diverges and consider the possibility that $\sum_l z\, \partial n(l)/\partial z$ diverges at the transition. Summing Eq. (11), supposing that $\varphi_2(0, 0)$ exists and has a finite nonzero value at the transition, and neglecting all non-

divergent contributions, we have

$$\sum_l z\,\partial n(l)/\partial z = \sum_l n^2(l)[1+\varphi_1(0) + \sum_{l'}\varphi_2(0,l')\,z\,\partial n(l')/\partial z + \varphi_2(0,0)\sum_l z\,\partial n(l')/\partial z], \quad (12)$$

where $\sum_{l'}$ implies summation over all l for which $|l|>\Delta$. Equation (12) is a contradiction under the hypotheses, since on the right we have a divergent quantity and on the left the same divergent quantity multiplied by another. In the case that $\sum_l n^2(l)$ converges $\sum_l z\,\partial n(l)/\partial z$, and therefore $z(\partial N/\partial z)$, converges a fortiori. By similar arguments all the second derivatives can be proved to converge.

In the case that $\sum_l n^2(l)$ diverges, an even more surprising conclusion can be drawn from Eq. (12). The only way this equation can be satisfied self-consistently is to give the left-hand side a finite nonzero value determined by setting the bracket on the right-hand side equal to zero:

$$\sum_l z\frac{\partial n(l)}{\partial z} = -[1+\varphi_1(0)+\sum_{l'}\varphi_2(0,l') \times z\frac{\partial n(l')}{\partial z}]/\varphi_2(0,0). \quad (13)$$

The above statement means nothing else than that $z[\partial n(l)/\partial z]$ as a function of l has a high and narrow shape around $l=0$ in the uncondensed state near the transition which becomes a δ-function at the transition. In the condensed state there is, of course, also a δ-function at $l=0$. Further computation yields the result that the two δ-functions and, indeed, the complete functions approach the same limit from above and below the transition. This fact has the consequence that $z(\partial N/\partial z)$ (and the other second derivatives) are continuous. The transition is at least of third order. Equation (12) also shows that an infinite value of $z(\partial N/\partial z)$ would result if $\varphi_2(0,0)$ were zero at the transition. This could be expected to be true at most at one point of the line $z = z_0(\beta)$.

The experiments of Fairbank et al.[3] have shown that C_p is infinite on the λ-line at the saturated vapor pressure and other authors[6,7] have found supporting evidence for the infinite value of the other derivatives at the same pressure. It has generally been assumed that this conclusion will be found to be the case all along the λ-line. Tisza[8] has recently pointed out that an extension of the phase rule presents us with the following dilemma: either the domain of the λ-line where the second derivatives are infinite is a point or there is a hitherto unsuspected degeneracy in the ground state of liquid helium. Tisza himself takes the second alternative. The above considerations suggest that further experiments should be carried out all along the λ-line to confirm or decisively rule out the first alternative.

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†On leave from the National Bureau of Standards, Washington, D. C.

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VLASOV INSTABILITY IN LONGITUDINAL PLASMA OSCILLATIONS

Peter L. Auer
Research Laboratory, General Electric Company, Schenectady, New York
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In a recent communication Buneman[1] raised the point that the well-known two-stream instabilities observed in electronic devices may have important consequences in plasmas of thermonuclear interest. For fully ionized plasmas the analog of the instabilities discussed by Buneman is to be found in the Vlasov equation, which describes the collective motions of the

plasma under the influence of externally applied plus internally generated coherent fields. The classic paper in this field is Landau's,[2] wherein it is shown that the evolution of plasma perturbations may be studied through an impulse response (or generalized impedance) function specified by the linearized version of the Vlasov equation. The spatial Fourier coefficient of this response function will be denoted here as $H_k(t)$, and the index k is always treated as real. In the present context the initial preparation of the plasma is stable or unstable according to whether there are any $H_k(t)$ which grow in time.

For an initially neutral plasma of uniform density, the $H_k(t)$ are given by the inverse Laplace transform of a quantity $1/D_k(p)$, where

$$D_k(p) = 1 + \frac{\omega_0^2}{ik}\int_{-\infty}^{\infty} \frac{f_0'(u)}{p+iku}\, du\,, \qquad (1)$$

$$\omega_0^2 f_0(u) = \sum \omega_\alpha^2 f_{0\alpha}(u)\,,$$

$$\omega_0^2 = \sum \omega_\alpha^2 = 4\pi \sum \frac{n_{0\alpha} e_\alpha^2}{m_\alpha}\,,$$

and $n_{0\alpha}$, e_α^2, and m_α are the partial initial density, charge, and mass of the α species in the plasma, and all $f_{0\alpha}$ are velocity distribution functions normalized to unity. The summations are over all charge species, subject to neutrality; the variable u denotes the velocity component in the direction of wave propagation. The important thing to observe is that $D_k(p)$ has a discontinuity along the entire Im(p) axis and its proper inversion requires that it be continued analytically from the positive into the negative real half-plane.[2,3] In the event the quantity $D_k(p)$ has no zeros in the complex plane, the above procedure assures that all $H_k(t)$ with finite k decay exponentially in time and the plasma remains stable. If zeros exist, they must come in conjugate pairs with positive and negative Re(p) of equal magnitude. The latter situation is unstable in that $H_k(t)$ can grow exponentially in time.

Since the nonlinear Vlasov equation conserves not only energy but also entropy,[4] it can be readily shown that all initial distributions Maxwellian in velocity are stable. Landau explicitly considers only these cases. More recently Berz[5] has shown that all bell-shaped distributions centered about the origin are stable. The analysis given by Berz is readily extended to show that any single-peaked composite distribution with or without an axis of symmetry is stable in the sense considered above. This follows from the lemma that $D_k(p)$ is analytic everywhere in the complex p plane (except for the discontinuity mentioned above) for real k and normalizable distribution functions with the property

$$f_0'(u) \gtrless 0 \text{ for } u \lesseqgtr u_0 \qquad (2)$$

for some u_0. The lemma is easily proved by the observation that when (2) applies, Re$D_k(p)$ is always greater than zero for Im$D_k(p)$ equal to zero.

Since the lemma given by (2) provides a sufficient criterion for stability, it immediately leads to a necessary condition for instability. Namely, it is necessary that the composite distribution function possess a minimum in order that unstable solutions in the sense considered here may arise. Although this requirement is not sufficient to assure that unstable solutions exist, it does allow us to compute lowest upper bounds on stability.

For example, an important consequence of (2) is the fact that an otherwise initially quiescent plasma containing a macroscopic current need not be unstable. Consider a plasma composed of singly charged positive ions of mass M and electrons at a uniform density n_0 and with initial current density J_0. If it is assumed that initially both ions and electrons have velocity distributions which are Maxwellian centered about their respective average stream velocities, the necessary condition for instability mentioned above leads to the conclusion that this plasma preparation is unconditionally stable in our sense provided J_0 satisfies the inequality

$$J_0 \leq e n_0 \gamma(\mu)(2kT/M)^{1/2}\,, \qquad (3)$$

$$\gamma(\mu) > 1\,;$$

where e is the magnitude of the electronic charge, k is Boltzmann's constant, T is the temperature, and μ is the ratio of ion to electron mass. For a deuterium plasma γ has the value of 2.8; with an initial density of 10^{15} particles per cc and temperature of 10 kv, the relation in (3) provides that the lowest upper bound on the current is 4000 amp/cm^2.

In view of the sizeable currents computed above one may be led to conclude that Vlasov instabilities of the simple type considered here will not play a major role in the problem of magnetic confinement and attendant diamagnetic currents. The nature of the problem, however, is appreciably altered by the presence of inhomogeneous magnetic fields and must be investigated separat

ly. On the other hand, the situation investigated here may have a valid application to the tenuous plasma of the sun's corona and possibly these simple types of Vlasov instabilities have some connection with solar prominences.

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PROTON GYROMAGNETIC RATIO

R. L. Driscoll and P. L. Bender
National Bureau of Standards,
Washington, D. C.
(Received November 12, 1958)

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In the final measurements with the apparatus the scatter was usually less than 1 ppm. Most of this was due to short-term variations in the earth's field and in the current through the solenoid. For protons in water the preliminary result that has been obtained in terms of the NBS electrical standards is $\gamma_p = (2.67515 \pm 0.00001) \times 10^4$ gauss^{-1} sec^{-1}. Benzene was found to give a result 1.9 ± 0.2 ppm higher. The accuracy of the result will be increased somewhat when further measurements on the solenoid pitch are available and when more checks on other possible sources of small systematic errors have been made. Measurements with a second precision solenoid on a Pyrex form are also under way.

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Recently Cohen and DuMond[10] have given corrections to their 1955 set of adjusted values of the atomic constants[11] for a revised theoretical estimate of the anomalous moment of the electron. However, if the present value of the proton gyromagnetic ratio were used instead of the value obtained by Thomas, Driscoll, and Hipple, the values of a number of the constants in the 1955 adjustments of Cohen, DuMond, Layton,

plasma under the influence of externally applied plus internally generated coherent fields. The classic paper in this field is Landau's,[2] wherein it is shown that the evolution of plasma perturbations may be studied through an impulse response (or generalized impedance) function specified by the linearized version of the Vlasov equation. The spatial Fourier coefficient of this response function will be denoted here as $H_k(t)$, and the index k is always treated as real. In the present context the initial preparation of the plasma is stable or unstable according to whether there are any $H_k(t)$ which grow in time.

For an initially neutral plasma of uniform density, the $H_k(t)$ are given by the inverse Laplace transform of a quantity $1/D_k(p)$, where

$$D_k(p) = 1 + \frac{\omega_0^2}{ik}\int_{-\infty}^{\infty}\frac{f_0'(u)}{p+iku}\,du\,, \qquad (1)$$

$$\omega_0^2 f_0(u) = \sum \omega_\alpha^2 f_{0\alpha}(u)\,,$$

$$\omega_0^2 = \sum \omega_\alpha^2 = 4\pi \sum \frac{n_{0\alpha} e_\alpha^2}{m_\alpha}\,,$$

and $n_{0\alpha}$, e_α^2, and m_α are the partial initial density, charge, and mass of the α species in the plasma, and all $f_{0\alpha}$ are velocity distribution functions normalized to unity. The summations are over all charge species, subject to neutrality; the variable u denotes the velocity component in the direction of wave propagation. The important thing to observe is that $D_k(p)$ has a discontinuity along the entire Im(p) axis and its proper inversion requires that it be continued analytically from the positive into the negative real half-plane.[2,3] In the event the quantity $D_k(p)$ has no zeros in the complex plane, the above procedure assures that all $H_k(t)$ with finite k decay exponentially in time and the plasma remains stable. If zeros exist, they must come in conjugate pairs with positive and negative Re(p) of equal magnitude. The latter situation is unstable in that $H_k(t)$ can grow exponentially in time.

Since the nonlinear Vlasov equation conserves not only energy but also entropy,[4] it can be readily shown that all initial distributions Maxwellian in velocity are stable. Landau explicitly considers only these cases. More recently Berz[5] has shown that all bell-shaped distributions centered about the origin are stable. The analysis given by Berz is readily extended to show that any single-peaked composite distribution with or without an axis of symmetry is stable in the sense considered above. This follows from the lemma that $D_k(p)$ is analytic everywhere in the complex p plane (except for the discontinuity mentioned above) for real k and normalizable distribution functions with the property

$$f_0'(u) \gtrless 0 \text{ for } u \lesssim u_0 \qquad (2)$$

for some u_0. The lemma is easily proved by the observation that when (2) applies, Re$D_k(p)$ is always greater than zero for Im$D_k(p)$ equal to zero.

Since the lemma given by (2) provides a sufficient criterion for stability, it immediately leads to a necessary condition for instability. Namely, it is necessary that the composite distribution function possess a minimum in order that unstable solutions in the sense considered here may arise. Although this requirement is not sufficient to assure that unstable solutions exist, it does allow us to compute lowest upper bounds on stability.

For example, an important consequence of (2) is the fact that an otherwise initially quiescent plasma containing a macroscopic current need not be unstable. Consider a plasma composed of singly charged positive ions of mass M and electrons at a uniform density n_0 and with initial current density J_0. If it is assumed that initially both ions and electrons have velocity distributions which are Maxwellian centered about their respective average stream velocities, the necessary condition for instability mentioned above leads to the conclusion that this plasma preparation is unconditionally stable in our sense provided J_0 satisfies the inequality

$$J_0 \leq e n_0 \gamma(\mu)(2kT/M)^{1/2}\,, \qquad (3)$$

$$\gamma(\mu) > 1\,;$$

where e is the magnitude of the electronic charge, k is Boltzmann's constant, T is the temperature, and μ is the ratio of ion to electron mass. For a deuterium plasma γ has the value of 2.8; with an initial density of 10^{15} particles per cc and temperature of 10 kv, the relation in (3) provides that the lowest upper bound on the current is 4000 amp/cm^2.

In view of the sizeable currents computed above one may be led to conclude that Vlasov instabilities of the simple type considered here will not play a major role in the problem of magnetic confinement and attendant diamagnetic currents. The nature of the problem, however, is appreciably altered by the presence of inhomogeneous magnetic fields and must be investigated separate-

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and Rollett[11] and of Bearden and Thomsen[12] would be changed by about twice the quoted uncertainties. The values given should thus be regarded as uncertain to about this degree.

In view of the necessary choice of weights for the various measurements of the proton gyromagnetic ratio and the preliminary nature of the present result, it does not seem desirable to calculate improved values for the atomic constants at this time. It is interesting to note, however, that the present value of the gyromagnetic ratio, when combined with the average of the available proton cyclotron resonance results,[13] gives a value for the Faraday which is about half-way between the values obtained from the iodine and the silver voltameters.[11,12] A new complete adjustment of the constants should be delayed until a number of relevant experiments now under way are completed. At that time the value of the ampere as maintained by the national laboratories will probably be one of the quantities to be adjusted.

We would like to express our thanks to the many people at NBS who have contributed to this experiment. In particular we would like to thank L. Costrell who designed the electronic apparatus and R. D. Cutkosky and L. M. Branscomb who collaborated on the early part of the experiment. We are very much indebted to the U. S. Coast and Geodetic Survey for permission to carry out the experiment at the Fredericksburg Magnetic Observatory, and to Mr. R. E. Gebhardt and the members of the Observatory staff for their active cooperation. We should also like to thank M. Packard and A. L. Bloom of Varian Associates for valuable information on free-precession measurements in the earth's field.

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GAMMA-RAY BRANCHING AND SOME SPIN AND PARITY ASSIGNMENTS IN B^{11}

A. J. Ferguson, H. E. Gove, J. A. Kuehner, A. E. Litherland, E. Almqvist, and D. A. Bromley
Atomic Energy of Canada Limited, Chalk River, Ontario, Canada
(Received November 3, 1958)

Recent measurements have been made with the Chalk River electrostatic accelerator on the gamma-ray branching of the lower states of B^{11} populated in the reactions $Be^9(He^3,p\gamma)B^{11}$ [1] and $Li^7(\alpha,\gamma)B^{11}$.[2] Figure 1 shows the gamma-ray decay schemes obtained principally from the $Be^9(He^3,p\gamma)B^{11}$ reaction. In this set of experiments the gamma spectrum from each level was separated from the others by coincidences measured with the appropriate proton group. A small CsI proton counter resolved the proton groups to the levels up to the one at 8.92 Mev with the exception of the doublet at 6.76-6.81 Mev. This doublet has been resolved using a double-focussing "Kellogg" magnetic spectrometer. The gamma rays were detected in a 5 in. diam ×4 in.

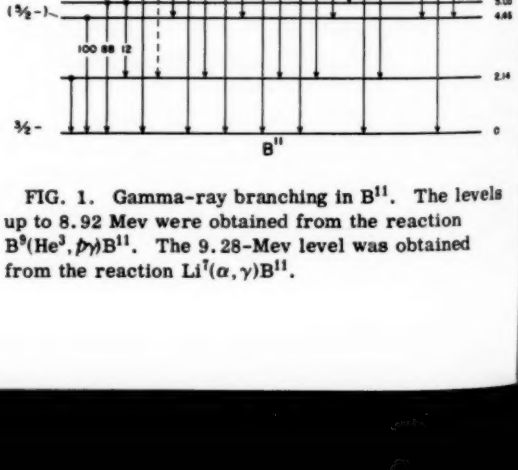

FIG. 1. Gamma-ray branching in B^{11}. The levels up to 8.92 Mev were obtained from the reaction $B^9(He^3,p\gamma)B^{11}$. The 9.28-Mev level was obtained from the reaction $Li^7(\alpha,\gamma)B^{11}$.

long NaI scintillator. An ion energy of 1.6 Mev was used for most of the He^3 studies. The branching of the 9.28-Mev state has been derived from the $Li^7(\alpha,\gamma)B^{11}$ reaction. The spectra were recorded on a 100-channel kicksorter and the intensities of the various components found by least-squares analysis using a Datatron electronic computer. The standard error assigned to the intensities in Fig. 1 is generally ± 6, although for the weaker transitions the errors are smaller than this. Particular attention was paid to the 6.76-Mev state. The use of the magnetic spectrometer to separate the proton group leading to this state from that of the 6.81-Mev state resulted in poor spectra with small numbers of counts. However, it was established that the 2.30-Mev primary to the 4.46-Mev state is definitely present. An upper limit of 8% is placed on the intensity of the 2.14-Mev component.

Identification of the branching of the 9.28-Mev level through the 6.76-6.81 Mev doublet is essential for the interpretation of angular distributions involving these states. Spectra taken with the $Li^7(\alpha,\gamma)B^{11}$ reaction at this level and expanded in the region of 2.5 Mev show that the cascades through this doublet proceed principally or wholly through the 6.76-Mev member. This is demonstrated by the following two observations:

1. The primary component has an energy of 2.52 Mev rather than 2.47 Mev. This is in agreement with the result of Jones and Wilkinson.[3]

2. A 2.30-Mev component is clearly present but there is no evidence of a 2.14-Mev component. This is compatible with the decay scheme observed for the 6.76-Mev state.

The recent work of Bilaniuk and Hensel[4] shows that the 9.28-Mev level has spin and parity either 5/2+ or 7/2+ with the former preferred. This choice is supported by the presence of the ground-state transition which would be $M2$ for the 7/2+ assignment. The angular distributions of the 9.28-Mev, 6.76-Mev, 4.82-Mev, 4.46-Mev, and 2.52-Mev gamma rays from the reaction $Li^7(\alpha,\gamma)B^{11}$ at this level have been measured.[2] The results for the 9.28-Mev and 4.82-Mev components are in excellent agreement with the assignments 5/2+ and 5/2- for the 9.28-Mev and 4.46-Mev states with pure P-wave formations. The angular distribution of the 4.46-Mev gamma ray indicates some $E2$-$M1$ mixing. The work of Cox and Williamson[5] indicates that the 6.76-Mev state has negative parity. The results for the 2.52-Mev and 6.76-Mev components exclude the assignment 5/2- for this level. If it is assumed to have the assignment 3/2- then the angular distribution of the 2.52-Mev component will be identical with the 9.28-Mev ground-state transition regardless of orbital mixing in the incoming alpha beam. It is found that the ratios of the second-order Legendre polynomial coefficient to the zero-order coefficient for the two radiations differ by twice the standard deviation. On the other hand the ratio for the 2.52-Mev component agrees with the theoretical one for a 7/2- assignment to well within the standard deviation. The latter assignment is thus strongly favoured by the data. The $(p\gamma)$ correlations measured by Cox and Williamson in the stripping reaction $B^{10}(d,p\gamma)B^{11}$ are additional evidence against the 3/2- assignment. The $Li^7(\alpha,\gamma)B^{11}$ angular distributions found here agree generally with those of Jones and Wilkinson[6] and of Meyer-Schützmeister and Hanna.[7]

Many of the branching ratios are in excellent agreement with the theoretical values of Kurath[8] if the spin-orbit coupling parameter a/K has a value near 4.5. In particular the $M1$ matrix element for the $7/2- \rightarrow 5/2-$ transition vanishes so that the strong $E2$ transition of the 6.76-Mev state to ground compared with the $M1$ transition to the 4.46-Mev state is quite reasonable. The $E2/M1$ amplitude ratio of the 4.46-Mev gamma ray deduced from its angular distribution is 0.14 ± 0.01 compared with 0.21 from Kurath's work. However, since the theoretical phase of the amplitude is not given there, this aspect of the agreement has not been fully investigated. The branching of the 5.03-Mev and 7.30-Mev states agrees excellently with the tentative assignments 3/2-* and 5/2-*, respectively, the asterisk indicating the second state of this spin and parity as in Kurath.

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RANGE OF THE SPIN-ORBIT FORCE BETWEEN TWO PROTONS*

P. S. Signell and R. Zinn
Bucknell University,
Lewisburg, Pennsylvania

and

R. E. Marshak
University of Rochester,
Rochester, New York
(Received November 3, 1958)

Gammel, Christian, and Thaler[1] were the first to carry through an extensive analysis of the two-nucleon scattering data up to 300 Mev on the basis of arbitrary combinations of central and tensor Yukawa-type forces. When this work failed to yield satisfactory agreement with experiment, two of the present authors[2] decided to try adding a phenomenological spin-orbit force to the particular combination of central and tensor forces defined by the meson-theoretic Gartenhaus potential. The rationale of this hybrid mixture has been discussed in the original papers[2] and in a paper by Okubo and one of the authors.[3] This so-called semiphenomenological SM potential gives a good fit to all of the two-nucleon data up to 150 Mev and becomes progressively worse as the energy is increased to 300 Mev. At least three questions must be raised: (1) How unique is the spin-orbit potential?[4] (2) How sensitive are the predictions to changes in the cores of the Gartenhaus potential?[5] (3) Can the scattering data at 300 Mev be explained by modifying the shape of the spin-orbit force and/or the cores of the Gartenhaus potential or must a higher-order velocity-dependent potential (e.g., the quadratic spin-orbit force[3]) be introduced? Some calculations giving partial answers to these three questions with regard to proton-proton scattering are reported in this note.

It was realized at the start that the Goldfarb-Feldman[6] parameters for the spin-orbit potential were not in accord with meson theory but they were at hand and gave such a good fit to the data that they were used. It was also recognized that the Gartenhaus potential would have to be modified at small distances for at least the triplet odd states since an unphysical bound $^3P_2$ state[2] resulted from the strongly attractive central part of the Gartenhaus potential in these states. We have now investigated the effect of decreasing the range of the spin-orbit potential from our initial value of 1.07×10^{-13} cm to the "meson-theoretical" value of 0.7×10^{-13} cm; the spin-orbit potential is of the form

$$V_{LS} = (V_0/x)\frac{d}{dx}\left(\frac{e^{-2x}}{x}\right), \qquad (1)$$

where $x = \mu r$ and V_0(=21 Mev) is adjusted to fit[7] the p-p differential cross section at 45° (c.m.) and 150 Mev. In order to eliminate the bound $^3P_2$ state, we have modified the Gartenhaus triplet odd potential to include an infinitely repulsive core out to $x_c = 0.37$. The resulting potential will be referred to as the SM1 potential.

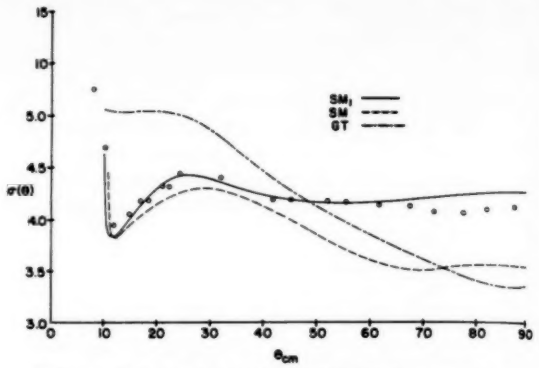

FIG 1. Comparison of 150-Mev proton-proton unpolarized cross section predictions with 147-Mev Harvard (8 and 14) experimental data.

The predictions of the SM1 potential for $\sigma(\theta)$, $P(\theta)$, and $D(\theta)$ at 150 Mev are shown in Figs. 1-3. Comparison is made with the predictions of the original SM and Gammel-Thaler[2] (GT) potential as well as with the experimental curves.[8] The SM and SM1 potentials make essentially the same predictions at energies lower than 150 Mev (e.g., 40 and 100 Mev[2]). At 300 Mev, it is interesting to compare the ^3P and ^3F phase shifts. In Table I we have listed these phase shifts corresponding to the SM1 and SM potentials and also the Stapp No. 1 set of phase shifts.[9]

From Figs. 1-3 and Table I, the following conclusions can be drawn: (1) At this stage, the spin-orbit force is not unique. Thus, a reduction in range — to match the "meson-theoretic" range — does not harm the agreement with the p-p scattering data at 150 Mev and even improves the agreement somewhat.[10] At 300 Mev, the reduced range of the spin-orbit potential yields phase shifts which constitute a distinct improvement over the previous ones. (2) Short-range repulsive cores can readily be introduced into the Gartenhaus potential to eliminate the unphysical bound states without adverse effects on the scattering.

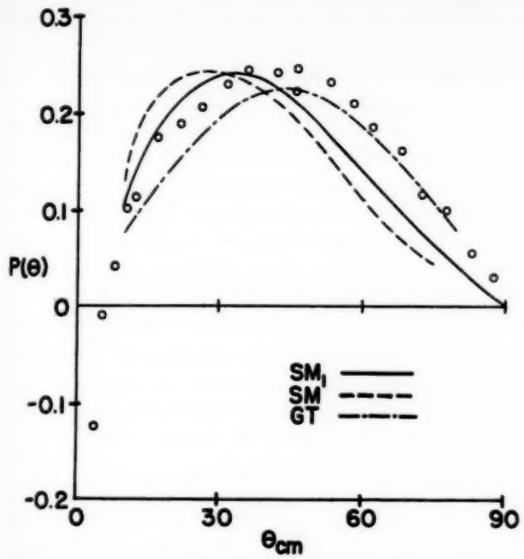

FIG. 2. Comparison of 150-Mev proton-proton polarization predictions with 147-Mev Harvard experimental data.

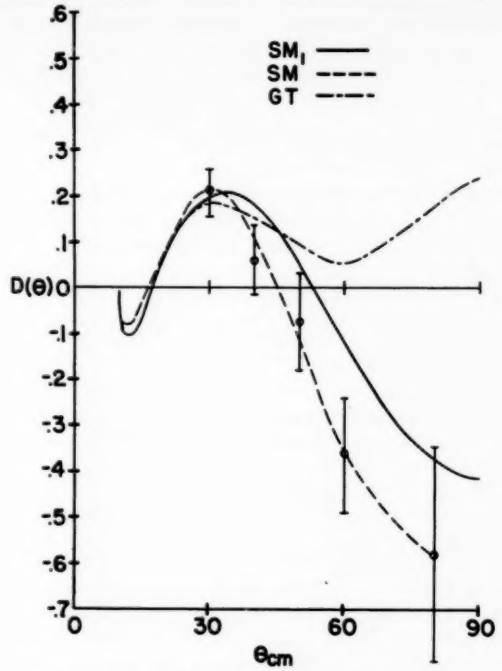

FIG. 3. Comparison of predictions for the triple scattering parameter $D(\theta)$ at 150 Mev.

(3) It is not excluded that a more exotic combination of spin-orbit and modified Gartenhaus potentials can explain the present p-p scattering data up to 300 Mev. A final decision as to whether the p-p interaction is more than linearly velocity-dependent in this energy region will probably require a complete determination of the experimental scattering matrix.[11]

We wish to thank Dr. L. Manchuck and Dr. R. Bliss of the IBM Data Processing Division at Endicott, New York, and Dr. T. A. Keenan, Director of the University of Rochester Computing Center, for their cooperation.

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[2]P. S. Signell and R. E. Marshak, Phys. Rev. 106, 832 (1957), and 109, 1229 (1958); this potential will be referred to as the SM potential. Independent calculations utilizing the spin-orbit force were carried out by J. Gammel and R. Thaler, Phys. Rev. 107, 291 (1957), and G. Breit, Phys. Rev. 111, 652 (1958).

[3]S. Okubo and R. E. Marshak, Ann. Phys. 4, 166 (1958).

[4]The meson-theoretic status of the spin-orbit potential is still obscure. It was shown in reference 3 that "p-wave" perturbation theory predicts an attractive spin-orbit potential (in the $I = 1$ state) with range $(1/2\mu)$ (μ is the pion mass) but with somewhat too small a magnitude. Recent dispersion-theoretic calculations by S. Okubo and S. Sato (private communication) have not improved the situation and have led the Japanese workers to question the necessity for introducing a spin-orbit force at all [S. Otsuki, Progr. Theoret. Phys. (Japan) 20, 171 (1958), and W. Watari, Progr. Theoret. Phys. (Japan) 20, 181 (1958)]. It is our belief that the present weight of evidence is strongly in favor of a significant spin-orbit force in the $I = 1$ (I is isotopic spin) state of the two-nucleon system; the evidence on the $I = 0$ state is at present inconclu-

Table I. Comparison of SM, SM1, and Stapp No. 1 nuclear (Blatt-Biedenharn) phase shifts in degrees.

300 Mev	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2	$^3F_2$	$^3F_3$	$^3F_4$
SM	9.8	-20.0	8.9	-20.8	-4.8	-4.9	7.4
SM1	- 1.2	-27.5	10.1	-20.1	-3.7	-4.7	6.5
Stapp No. 1 (310 Mev)	-14.3	-26.7	16.1	- 1.0	0.8	-4.4	3.1

sive [de Swart, Marshak, and Signell, Nuovo cimento 6, 1189 (1957)].

[5]It is to be recalled that the Gartenhaus potential [S. Garthenhaus, Phys. Rev. 100, 900 (1955)] is based on a static-nucleon source theory so that it cannot be trusted at short distances.

[6]L. Goldfarb and D. Feldman [Phys. Rev. 88, 1099 (1952)] took over the "Thomas-Yukawa" shape of the spin-orbit potential for which K. Case and A. Pais [Phys. Rev. 80, 203 (1950)] had previously given arguments.

[7]The fit is fairly sensitive to V_0: e.g., if $V_0 = 17.7$ Mev, $\sigma(45°) = 3.75$ mb/sterad and $P(37°) = 0.19$ mb/sterad at 150 Mev. For $V_0 = 23$ Mev, $\sigma(45°) = 4.51$ mb/sterad at 150 Mev.

[8]The experimental cross-section and polarization curves are taken from Palmieri, Cormack, and Wilson, Ann. Phys. (to be published). The D curve is from A. E. Taylor (private communication).

[9]The Stapp No. 1 phase shifts [Stapp, Ypsilantis, and Metropolis, Phys. Rev. 105, 302 (1957)] yield an excellent fit of all the p-p measurements at 310 Mev.

[10]It is interesting to note that the shorter range is also preferable if one relates the spin-orbit force in nuclei to the two-nucleon spin-orbit force; for example a range of $\approx 0.7 \times 10^{-13}$cm is needed to explain the $p_{3/2} - p_{1/2}$ splitting in He^5 [J. P. Elliot (private communication)].

[11]This requires the measurement of five independent quantities [see Puzikov, Ryndin, and Smorodinski, J. Exptl. Theoret. Phys. (U.S.S.R.) 5, 489 (1957), and reference 3].

PARITY AND OTHER SYMMETRIES IN STRONG INTERACTIONS

A. Pais
Institute for Advanced Study,
Princeton, New Jersey
(Received November 10, 1958)

In recent times, the question whether the high degree of P-conservation in nuclear phenomena precludes any P-violation in strong interactions has been on many peoples minds. In this respect one has especially thought of the virtual role of the new particles in nuclear interactions. We do not as yet have the theoretical tools to tackle such problems quantitatively. However, qualitative tests have been devised[1] to check P-conservation directly in hyperon reactions. A first application[2] to

$$\pi^- + p \to \Lambda + K^0 \qquad (1)$$

yields no evidence of appreciable P-violation in this strong production reaction. Of course, this valuable information does not settle all issues quite definitely. Here we wish to comment further on this problem. Briefly the idea is the following: it has been shown previously[3] how possible deviations from invariance laws for nucleon and pion systems could be masked to a large extent if the baryon meson system can approximately be assigned a rather high symmetry, the doublet approximation (DA). In this note we shall explore the question of P-violation from the same viewpoint. This will lead to two qualitative questions which can be put to many tests.

In the DA the S-number splits[4] into two parts S_1, S_2 which are separately conserved. Thus in the DA we have one more selection rule. This added constraint forbids a small number of reactions, notably

$$\pi^+ + p \to \Sigma^+ + K^+, \; K^- + p \to \Sigma^+ + \pi^-,$$
$$\pi^- + n \to \Sigma^- + K^0, \; \bar{K}^0 + n \to \Sigma^- + \pi^+; \qquad (2)$$

but π-nucleon transitions and reactions like (1) are not forbidden. Thus as long as the DA is P-conserving[5] we have to this approximation neither P-violation in π-nucleon systems, not even due to virtual K-effects, nor in reactions like (1). But what about the reactions (2)?

As has been noted before[3,4] the very fact that the reactions (2) are so inhibited in a DA means either than a DA is just no approximation at all, or else that a "doublet perturbation" (DP) is needed to break the (S_1, S_2)-rules. What would happen if the latter alternative were true and if the DP would break the P-conservation of the DA? If the DP would only feed the channels (2) we would say that the DP leads to P-violations in those and only those reactions. But the DP may also contribute to (S_1, S_2)-allowed reactions of type (1) and may therefore add P-violating to the P-conserving contributions of the DA. Nevertheless, it seems reasonable to raise two questions.

(I) Are the production and absorption reactions which would be blocked by the (S_1, S_2)-rules[6] P-conserving to the same extent as the other channels? A marked difference would indicate that a DA is useful.

It should be emphasized that, even so, the applicability of a DA would be worse where the (Σ, Λ) mass difference can least be neglected. This is most prominently the case for Σ + nucleon $\to \Lambda$ + nucleon exchange at low Σ energies: these reactions are not forbidden by the (S_1, S_2)-rules but may be strongly distorted by the DP, just as $\pi^- + p \to \pi^0 + n$ at zero π^- momentum is not for-

bidden by isotopic spin but is strongly modified by electromagnetic and mass-difference effects.

The second question is mainly relevant only if a DA were to exist. In order to formulate it, we have to trace back previous arguments a few steps.

If the reactions (2) were to show appreciable P-violation, the concepts of $\Sigma\Lambda$-parity, of relative parity $p(K)$ of charged and neutral K's, and of Ξ-nucleon parity may cease to be meaningful. However, up to the DA stage they are supposed to be well defined. Even so, there remains an ambiguity in the DA, namely whether $p(K)$ is even or odd.[3] In either case one can introduce a DP which violates the separate (S_1, S_2) conservation.[7] In the present spirit it could also violate P; but we shall require CP-invariance for it.[8] Further we may require that the DP conserves the conventional isotopic spin T. If the DA comprises all couplings linear in π and K to baryons, it is easily shown for either $p(K)$, that under the stated conditions the simplest DP is of the four-baryon types; it can have the (V, A) form. The influence of the DP on nucleon-nucleon interactions is then quite short ranged, as it comes about via virtual hyperon pairs.

In a previous study of the case of odd $p(K)$ we had explored the logical alternative for the DP: the simplest interaction was considered under the explicit assumption of P-conservation.[9] This coupling had to violate isotopic spin.

For odd $p(K)$, the present alternative – T-conservation, P-violation – would largely restore the isotopic spin conservation in K^--d reactions. It is easy to see that in this way no new deviations for the branching ratios in K^- absorption can occur[10] to the extent that the charged and neutral K-couplings of the DA do not interfere. Where then lies the test for odd $p(K)$? In general the most sensitive tests lie where one compares the reaction rates involving charged and neutral K particles. First of all this occurs in the verification of the triangle inequalities in (πp)-production reactions.[3] A more decisive test comes from situations where the possible oddness of $p(K)$ could produce a clash with charge symmetry: (II) Is charge symmetry valid in the comparison of production (or absorption) rates of charged and neutral K particles? Thus one asks for a comparison of K^+ production in (π^+, X) collisions with K^0 production in (π^-, X) collisions, where X is a self-conjugate nucleus like d, He. Or one compares channel by channel the absorption of K^- and $\bar{K}^0$ (or K_2^0) on X, etc.

To recapitulate: (I) can give further guidance concerning P-conservation; (II) is addressed to the structure of a possible DA.

Finally it may be observed that the possible existence of a DA would hold out a promise that CP-invariance is the truly universal reflection principle, since it has been shown that a DA is necessary[11] for extending symmetries to K couplings of such a nature that CP-invariance leads to separate conservation of C and P. Indeed, the requirement of minimal interactions only[12] and of CP-invariance provide sufficient constraints for restricting possible P-violations to the DP in addition to the weak interactions.

The author is much indebted to the members of the deuterium bubble chamber group at Berkeley, Dr. Horwitz, Dr. Miller, Dr. Murray, Dr. Schwartz, and Dr. Taft, for raising questions which stimulated the present comments. He would also like to acknowledge the benefit of many further discussions with Dr. M. Schwartz.

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[1] V. Soloviev, Nuclear Phys. 6, 618 (1958); Drell, Frautschi, and Lockett (to be published).

[2] Crawford, Cresti, Good, Solmitz, and Stevenson, Phys. Rev. Lett. 1, 209 (1958).

[3] A. Pais, Phys. Rev. 112, 624 (1958). The invariance specifically discussed in this paper is the charge independence of π-nucleon systems.

[4] A. Pais, Phys. Rev. 110, 574 (1958). The quantum numbers S_1, S_2 mentioned below are defined in Table I of this paper.

[5] It is not impossible to formulate a DA with indefinite K-parities. But then one cannot strictly require CP-invariance to hold in the presence of all electromagnetic interactions.

[6] Also various Ξ-reactions are of this type: $\pi^- + n \to \Xi^- + 2K^0$, $\pi^+ + n \to \Xi^- + 2K^+$, $K^- + p \to \Xi^- + K^+$, etc. Note also the possibility of using $\pi^- + p \to \Sigma^- + K^0 + \pi^+$; $K^+ + p \to K^0 + p + \pi^+$; etc. for measuring up-down asymmetries in production. The importance of K-exchange scattering, forbidden by (S_1, S_2) conservation, becomes emphasized once more.[3]

[7] It is sufficient that this interaction couples charged to neutral K particles and/or N_2 to N_3 states, as defined in reference 4, Eqs. (14), (15).

[8] This guarantees a null electric dipole moment for the neutron, see L. Landau, Nuclear Phys. 3, 127 (1957). For experimental aspects see Smith, Purcell, and Ramsey, Phys. Rev. 108, 120 (1957).

[9] This is the $KK\pi$-interaction of reference 3, Sec. IV.

[10] See also reference 3, Sec. VI, remark 2.

[11] But not sufficient; further degeneracies are necessary. These questions are discussed by J. Sakurai, University of California Radiation Laboratory Report UCRL-8440 (to be published); G. Feinberg and F. Gürsey (to be published). These papers contain full

references to earlier work on *CP*-invariance.

[12]That is, fermion-fermion-boson interaction without derivatives. For the bearing of this condition on electromagnetism, see Ia. Zel'dovitch, J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1531 (1957) [translation: Soviet Phys. JEPT 6, 1184 (1958)].

ERRATUM

OPTICAL MODEL OF INELASTIC SCATTERING. T. K. Fowler [Phys. Rev. Lett. 1, 371 (1958)].

In Table I, the number in the fourth line of column 2 should be 12.5 instead of 12.6.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

QUANTUM THEORY OF SURFACE ENERGY AND TENSION. R. Brout and M. Nauenberg, Department of Physics, Cornell University, Ithaca, New York (Received August 4, 1958).

The surface energy E_S of a system of interacting particles is $E_S = \int_{-\infty}^{\infty} dz[\epsilon(z) - \epsilon_B \rho(z)/\rho_B]$ where $\epsilon(z)$, $\rho(z)$ are the energy and particle densities along the axis normal to the surface, ϵ_B, ρ_B are the corresponding quantities in the bulk. Alternatively one may formulate an exact expression for the surface tension defined as $\gamma = (\partial F/\partial S)_V$ with the result that

$$\gamma = \int_{-\infty}^{\infty} dz \{ 2[t_z(z) - t_\perp(z)] + \tfrac{1}{4}\int d^3r(r^2 - 3r_z^2) \times (1/r)(dv/dr)\rho^{(2)}(z, z + r_z).$$

$t_i(z)$ are the components of the average kinetic energy density and $\rho^{(2)}(r_1 r_2)$ is the pair distribution function. In the ground state $\gamma = E_S$ we have applied these formulas to liquid He^4 and He^3 assuming that $\rho^{(2)} = \rho(z)\rho(z + r_z) g_B(r)$. The bulk radial distribution $g_B(r)$ is obtained from x-ray data. Use of the thermodynamic data ϵ_B and $p = 0$ gives t_B and checks $g_B(r)$. Further we assume a free volume form of the kinetic energy density $t(z) = t_B[\rho(z)/\rho_B]^{5/3}$ and an exponential density fall-off, $\rho(z) = \rho_B \epsilon^{-z/a}$. Calculated values of E_S show a minimum at $a = 2A$. E_S and γ differ by 20% at this value with γ being 0.38 erg/cm² for He^4. The experimental value of γ is 0.35 erg/cm².

TRANSVERSE PLASMA WAVES AND PLASMA VORTICES. O. Buneman, Stanford Electronics Laboratories, Stanford University, Stanford, California (Received April 2, 1958).

Plasmas at high electron temperatures can carry transverse waves in which self-magnetic fields and relativistic effects become important. In this paper the relativistic perturbation equations for an isotropic uniform plasma are solved as an initial value problem, i.e., by Laplace transformation, and the propagation or dispersal of both longitudinal and transverse perturbations is calculated. In both cases transients occur which have a continuous frequency spectrum. While transverse perturbations also yield pure persistent waves (with phase velocity exceeding that of light) of all wavelengths, longitudinal perturbations of very short wavelengths will not be propagated as pure waves but will die out eventually with only longer wavelengths persisting. The transverse plasma perturbations discussed in the analysis are non-vortical and the dispersal of vortices is covered by a separate discussion. The vortices do not give rise to a new mode of propagation of perturbations.

CONDUCTIVITY OF PLASMAS TO MICROWAVES. Edward A. Desloge, Steven W. Matthysse and Henry Margenau, Laboratory of Marine Physics, Yale University, New Haven, Connecticut (Received August 4, 1958).

If one assumes that the effect of an alternating electric field on an electron in a slightly ionized gas is on the average an addition, in the direction of the field, of a velocity component, the magnitude of which is dependent on the speed of the electron, then it is possible to derive very simply the expression for conductivity obtained earlier by Margenau. In addition an alternative expression is proposed which is of interest because it avoids difficulties with negative conductivities encountered by Margenau's formula.

APPROACH OF ONE-DIMENSIONAL SYSTEMS TO EQUILIBRIUM. Joseph Ford, Metals Research Laboratories, Electro Metallurgical Company, Niagara Falls, New York (Received August 7, 1958).

This paper defines an equilibrium coarse-grained probability density $F(\vec{X}, \vec{P}, \infty)$ in terms of the Liouville fine-grained density $f(\vec{X}, \vec{P}, t)$ through the equation

$$F(\vec{X}, \vec{P}, \infty) = \lim_{\Delta\Omega \to 0} (1/\Delta\Omega) \lim_{t \to \infty} \int_{\Delta\Omega} d\vec{X} d\vec{P} f(\vec{X}, \vec{P}, t)$$

and points out that $F(\vec{X}, \vec{P}, \infty)$ is the density which determines the observable properties of a system at equilibrium. Given a sufficiently smooth initial density $f_0(\vec{X}_0, \vec{P}_0, 0)$, $F(\vec{X}, \vec{P}, \infty)$ is shown to exist and describe the equilibrium behavior of two one-dimensional systems. This equilibrium behavior is occasioned by the presence of non-

linear forces. Because of the nonlinearity, the Poincaré recurrence time, for a given accuracy of return, depends on initial conditions. It is shown that this dependence causes a Gibbs type stirring of phase space which leads to equilibrium.

VARIATIONAL METHODS FOR SCATTERING PROBLEMS. Marius Kolsrud, Institute for Toeoretical Physics, University of Oslo, Blindern, Norway (Received August 4, 1958).

Some new stationary expressions for the phase shifts and total amplitude by central force scattering are presented. As examples values of $\tan\delta_0$ are given for the exponential and Yukawa potential, and compared with the results of other methods and with the exact values.

LOW-TEMPERATURE BEHAVIOR OF A DILUTE BOSE SYSTEM OF HARD SPHERES I. EQUILIBRIUM PROPERTIES. T. D. Lee* and C. N. Yang,† Brookhaven National Laboratory, Upton, New York (Received August 13, 1958).

By a generalization of the method used in a previous paper, the distribution of energy levels of a dilute system of Bose hard spheres is found. These energy levels are then used to compute the statistical properties for the system. A phase transition is found, and the transition point is calculated to the lowest order of a, the diameter of the hard spheres. Furthermore, the thermodynamical functions of the system in both the gas phase and the degenerate phase are obtained, provided $(a/\lambda) \ll 1$ and $\rho a^3 \ll 1$ where λ is the thermal wave length and ρ is the particle density.

* Permanent address: Columbia University, New York, New York.

† Permanent address: The Institute for Advanced Study, Princeton, New Jersey.

SUPRATHERMAL PARTICLES III. ELECTRONS. E. N. Parker, Enrico Fermi Institute for Nuclear Studies and the Department of Physics, University of Chicago, Chicago, Illinois (Received July 25, 1958).

Certain phenomena in nature, such as the 50- to 100-kev electrons of the aurora, suggest that there are plasma-dynamical processes which can quickly transfer the translational energy of the ions in a plasma stream to the electrons (some 20 kev/ion for a 2000-km/sec solar wind). It is shown that two interpenetrating streams of noncolliding and initially neutral plasma can achieve this energy transfer with a characteristic time comparable to $(M/m)^{1/2}$ times the plasma period. The process is closely analogous to the excitation of plasma oscillations by two interpenetrating electron streams, but of course proceeds to much greater electron energies because the ion components of the streams carry so much more kinetic energy than do the electron components. Hence, besides the auroral electrons, it is probably responsible for solar radio emission rather than the electron streams implied in current theories.

Further, the process is probably the dominant interaction in shock fronts, particularly in astrophysical cases where neither direct collisions nor the existing weak magnetic fields can give sharp fronts. The characteristic thickness of a shock front in the solar wind is of the order of 10^4 cm due to the above plasma excitation whereas the Larmor radius of the ions, which would otherwise determine the shock thickness, in the absence of collisions, is 100 km or more.

KINEMATICS OF GROWING WAVES. P. A. Sturrock,* Microwave Laboratory, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, California (Received July 11, 1958).

This paper is concerned with the problem of distinguishing between amplifying and evanescent waves. These have, in the past, been distinguished by considerations of energy transfer or of the initial and boundary conditions with which a wave must be associated. Both procedures are open to criticism.

The problem is here interpreted kinematically: we investigate the classes of wave functions which a given propagating system may support, without inquiring into the way these disturbances may be set up, and postponing inquiry into the boundary conditions necessary. In this way, we may distinguish between amplifying and evanescent waves by determining whether a wave function which may be analyzed into "real-frequency" waves may also be analyzed into "real-wave-number" waves. This question may be answered by means of a certain diagram, which may be constructed from knowledge of the dispersion

relation.

Interchange of the roles of time and space leads to the statement and solution of a further problem. If a propagating system is unstable, the instability may be such that a disturbance grows, but is propagated away from the point of origin: this is termed "convective instability". On the other hand, the instability may be such that the disturbance grows in amplitude and in extent, but always embraces the original point of origin: this is termed "nonconvective instability". The statement that the system supports amplifying waves is synonymous with the statement that the system exhibits convective instability. A system which exhibits nonconvective instability may not be used as an amplifier, but may be used as an oscillator. It is possible to distinguish between convective and nonconvective instability by a further diagram which also may be constructed from knowledge of the dispersion relation.

Our theory enables us to make the following assertions. If ω is real for all real k, then any complex k, for real ω, denotes an evanescent wave. Conversely, if k is real for all real ω, then any complex ω, for real k, denotes nonconvective instability.

*At present Visiting Scientist, European Council for Nuclear Research, Geneva.

THERMAL EXCITATIONS IN LIQUID He^3. Louis Goldstein, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received May 1, 1958).

Using recently obtained data on various thermal properties of liquid He^3, the constant-volume heat capacity of the saturated liquid has been evaluated. This elementary heat capacity $C_v(T)$ was found to exhibit at least three inflection points over the temperature interval extending to the critical temperature. Of these, the one at the lowest temperature was shown to originate with the partial heat capacity of spin disorder. The inflection point at the approach of the critical temperature T_c is imposed by the thermodynamic result that $C_v(T)$ should reach its finite limit $C_v(T_c)$ from below with positively infinite first and second temperature derivatives. The laws governing the pressure dependence of the partial spin heat capacity over the liquid region of the He^3 phase diagram have been deduced. The remarkable pressure effects displayed by the spin system of liquid He^3 through its heat capacity may be expected to manifest themselves through the observable pressure dependence of the total liquid heat capacity at low enough temperatures where the spin heat capacity is dominant. A purely heuristic and indirect approach toward the problem of the existence of phonon type of partial excitations in liquid He^3 yields a negative answer, without, however, a definite exclusion of such symmetrical excitations in this antisymmetric liquid.

VOLUME ANOMALY OF LIQUID He^3 ARISING FROM ITS NUCLEAR SPIN SYSTEM. Louis Goldstein, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received May 26, 1958).

The anomalous volume expansion coefficient of liquid He^3 derived in a previous work through a discussion of its nuclear spin system will be shown to be describable quantitatively within the statistical thermodynamic formalism attached to its nuclear spin degrees of freedom. The theory predicts, through the large partial negative spin expansion coefficient, a volume minimum or a density maximum of saturated liquid He^3 subject to the volume behavior determined by its nonspin degrees of freedom. The temperature at which the density maximum should occur is higher than about 0.2°K. At this temperature, or very close to it, the negative spin expansion coefficient has a sharp minimum and the total liquid density is expected to have an inflection point in the density-temperature representative plane. Since the whole anomaly depends directly on the liquid compressibility, it should be most pronounced in the saturated liquid and should decrease under the application of external pressure.

PLASMA OSCILLATIONS WITH DIFFUSION IN VELOCITY SPACE. A. Lenard and Ira B. Bernstein, Project Matterhorn, Princeton University, Princeton, New Jersey (Received August 12, 1958).

A model of plasma oscillations in the presence of small-angle collisions is presented which admits of exact analytic solution. Certain features of the true collision terms are preserved. Namely, the effect of collisions is represented

by a diffusion in velocity space, which makes the distribution function tend to the Maxwell distribution, and which conserves the number of particles. In the limit of infrequent collisions, the results of Landau are recovered.

LORENTZIAN GAS AND HOT ELECTRONS. I. Adawi, RCA Laboratories, Princeton, New Jersey (Received April 24, 1958; revised manuscript received August 14, 1958).

The problem of hot electrons in a nonpolar crystal is reconsidered using the Lorentzian gas model more accurately. Scattering by acoustical phonons alone is considered first. The new results are (1) an asymptotic formula for the moments of the velocity distribution which permits calculation of the deviation from the square root law at high fields; (2) a recursion system allowing the calculation of any velocity polynomial in terms of the average energy, random velocity, and mobility of the electrons. Scattering by ionized impurities in addition to acoustical phonons is considered next and the distribution function is derived. The proportionality constant relating the change in the low-field mobility to E^2 is shown to be highly sensitive to ionized impurity scattering. Thus, appreciable changes from its value for pure lattice scattering occur for μ_0/μ_I as low as 10^{-3}. (E is the field strength and μ_0 and μ_I are the low-field lattice and impurity mobilities, respectively.) It is pointed out that substantial deviations from results obtained using a Maxwellian distribution do occur.

INFLUENCE OF MASS RATIO ON DISPLACEMENT CASCADES IN DIATOMIC SOLIDS. E. M. Baroody, Battelle Memorial Institute, Columbus, Ohio (Received July 18, 1958).

The influence of mass ratio on the displacement cascade initiated when an atom of a solid of type AB receives considerable energy is investigated for a simple model. Isotropic scattering, equal probabilities for collisions of various pairs of atoms, and a single threshold energy for displacement are assumed. The formulation of the problem is like that of Harris, but threshold energy is introduced in the manner of Kinchin and Pease. Comparatively simple equations are obtained and solutions which are always approximately correct have been found by the Laplace transform method.

The total number of displacements does not vary greatly with mass ratio. When the mass of the heavy atom is more than about ten times that of the light the difference between the numbers of A atoms and B atoms displaced by a primary A atom increases as a fractional power of the primary energy. For smaller mass ratios, the difference approaches a constant. To illustrate some implications of the results, the relative numbers of A and B atoms displaced by incident monoenergetic protons are considered.

TWO HALL EFFECTS OF IRON COBALT ALLOYS. Frank P. Beitel, Jr., and Emerson M. Pugh, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received August 4, 1958).

The two Hall coefficients and the resistivity of Fe-Co alloys have been measured at 77°K, 169°K, and room temperature, using fields up to 3.3 weber/m^2. For up to 0.2% Co in Fe the ordinary Hall coefficient R_0 is positive at room temperature but becomes negative at low temperatures; for all other compositions R_0 is negative. Analysis with a simple model in which the (4s) conduction band consists of a parallel and an antiparallel sub-band shows that, for up to 20% Fe in Co, the parallel 3d sub-band is filled. It then empties as the Fe content increases, having about 0.2 holes per atom at 50% Fe, but at 65% Fe the bands apparently shift so that again the parallel 3d sub-band is filled. These conclusions remain unchanged if the analysis is based instead upon a model proposed recently in which the 3d electrons may be in nonconducting states as well as in the usual conducting-type states. The extraordinary Hall coefficient R_1 is positive for less than 25% Co in Fe, but for all other compositions it changes from positive to negative as the temperature decreases from 300°K to 169°K. R_1 and the resistivity ρ satisfy the relation $R_1 = a + b\rho^2$.

SOME LOW-TEMPERATURE PROPERTIES OF THE R, M, AND N CENTERS IN KCl AND NaCl. W. Dale Compton and Clifford C. Klick, United States Naval Research Laboratory, Washington, D. C. (Received August 13, 1958).

Temporary changes in the absorption spectrum of a colored NaCl crystal can be produced at 77°K and 4°K by irradiating the crystal with a strong auxiliary light. The changes in the ab-

sorption spectrum indicate that the R, M, and N centers are not randomly distributed but are located near each other. It is concluded that previous arguments based upon less complete observations are not valid and that one cannot decide from this type of experiment whether the R_1 and R_2 bands arise from transitions in two distinct centers or from two transitions in the same center. Measurements of the degree of polarization of the emission excited with R_1- or R_2-band light indicate that the concentration of other centers probably affects this number. It is concluded that this measurement may not determine the symmetry of these centers. Measurements of the temperature dependence of the half-widths of the absorption and emission bands of the M center indicate that the optical transition is primarily influenced by local interactions of the center with neighboring ions and not by interactions with the long range phonons of the lattice.

THEORY OF CYCLOTRON RESONANCE IN METALS. Sergio Rodriguez,* Department of Physics, University of California, Berkeley, California (Received August 5, 1958).

The surface impedance of a metal, in the extreme anomalous skin effect region and in the presence of a magnetic field $\vec{H}_0$ parallel to its surface, is calculated assuming specular reflection and spherical energy bands. We give the surface impedance in the cases in which the microwave electric field is parallel and at right angles with H_0, which we call longitudinal and transverse respectively. The position, intensities, and width of the cyclotron resonance lines are the same for longitudinal and transverse cyclotron resonance.

*Present Address: Department of Physics, University of Washington, Seattle, Washington.

MAGNETIC RESONANCE LINE SHAPES AT THE ONSET OF SATURATION. D. F. Holcomb, Department of Physics, Cornell University, Ithaca, New York (Received August 11, 1958).

Magnetic resonance absorption line shapes and widths in Li metal and CaF_2 crystals have been measured as a function of radio-frequency power level. The measurements have been made on the Li^7 resonance in Li at 77°K and 215°K and on the F^{19} resonance in CaF_2 at room temperature. The measurements have been made from low rf power into the region of appreciable saturation, but not into the very high saturation region considered by Redfield. A decrease in δH, the peak-to-peak absorption derivative line width, was observed in both systems at sufficiently large values of H_1, the rf magnetic field. The changes of line width in Li at various values of T_1 and of characteristic motional frequency of the nuclei show good qualitative agreement with the criteria proposed in the recent theory of Tomita.

Line-width data obtained by Redfield in aluminum are included for comparison. In the case of CaF_2, where the spin-lattice relaxation proceeds via paramagnetic impurities, the decrease in δH occurs at appreciably higher saturation levels than is the case in Li and Al.

PIEZORESISTANCE IN n-TYPE GaAs. A. Sagar, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, and University of Pittsburgh, Pittsburgh, Pennsylvania (Received August 11, 1958).

Piezoresistance and elastoresistance coefficients of n-type GaAs were determined at room temperature. The results are consistent with a spherical energy band model as predicted by Callaway from theory.

MULTIPLE QUANTUM TRANSITIONS IN PARAMAGNETIC RESONANCE. P. P. Sorokin, I. L. Gelles, and W. V. Smith, Research Laboratory, International Business Machines Corporation, Poughkeepsie, New York (Received August 15, 1958).

At high rf fields the normal paramagnetic resonance spectrum of Mn^{++} in cubic MgO is found to be modified by the appearance of additional absorption lines which occur at the frequencies $(E_k - E_{k-2})/2h$. These are interpreted as double quantum transitions ($\Delta M = 2$) which occur between nearly equally spaced energy levels at high rf power. The transitions $M = -\frac{3}{2} \rightarrow +\frac{1}{2}$ and $M = -\frac{1}{2} \rightarrow \frac{3}{2}$ have been observed at both room and liquid nitrogen temperatures for a number of orientations of the crystal with respect to a dc magnetic field of 3400 oersteds. At 77°K evidence of the triple quantum transitions $-\frac{1}{2} \rightarrow \frac{5}{2}$, $-\frac{5}{2} \rightarrow \frac{1}{2}$ was observed.

Second order time-dependent perturbation theory yields a formula for the ratio of the absorption intensity of the double quantum transition to the absorption intensity for a first-order transition. The predicted ratio agrees with the experimentally observed ratio.

EVIDENCE FOR ANTIFERROMAGNETISM IN $Cu_3(CO_3)_2(OH)_2$. R. D. Spence and R. D. Ewing, Department of Physics, Michigan State University, East Lansing, Michigan (Received August 1, 1958).

We have used the proton resonance to study the magnetic behavior of the mineral azurite, $Cu_3(CO_3)_2(OH)_2$. Below 30°K the temperature dependence of the splitting of the proton resonance lines by the copper ions shows strong exchange effects. At 1.86°K a transition takes place and the resonance pattern below the transition indicates an antiferromagnetic state.

ANTIFERROMAGNETISM OF $CuF_2 \cdot 2H_2O$. G. S. Verma and K. Tokunaga, Western Reserve University, Cleveland, Ohio (Received August 13, 1958).

The perpendicular and parallel magnetic susceptibilities have been calculated for $CuF_2 \cdot 2H_2O$ on the basis of Nakamura's theory. The computed values of molar susceptibility for the same compound have been compared with the recent measurements of Bozorth and Nielsen and are found to be in good agreement.

RANDOM-PHASE APPROXIMATION IN THE THEORY OF SUPERCONDUCTIVITY. P. W. Anderson, Bell Telephone Laboratories, Murray Hill, New Jersey (Received July 28, 1958).

A generalization of the random-phase approximation of the theory of Coulomb correlation energy is applied to the theory of superconductivity. With no further approximations, it is shown that most of the elementary excitations have the Bardeen-Cooper-Schrieffer energy gap spectrum, but that there are collective excitations also. The most important of these are the longitudinal waves which have a velocity $v_F[\frac{1}{3} \times (1 - 4N(0)\,|\,V\,|)]^{\frac{1}{2}}$ in the neutral Fermi gas, and are essentially unperturbed plasma oscillations in the charged case. Other collective excitations resembling higher bound pair states may or may not exist but do not seriously affect the energy gap. The theory obeys the sum rules and is gauge invariant to an adequate degree throughout.

TRANSPORT IN METALS: EFFECT OF THE NONEQUILIBRIUM PHONONS. M. Bailyn, Department of Physics, Northwestern University, Evanston, Illinois (Received March 10, 1958).

The coupled Boltzmann equations for phonons and electrons in a metal are solved simultaneously using first-order perturbation theory. Use is made of the Kohler variational principle to put the transport coefficients in terms of determinants. The final results are in the form of integrals involving the phonon-phonon relaxation time $\tau_{\vec{q}}$ and the electron-electron relaxation time. The details of how a finite electrical conductivity exists by means of umklapp processes even when $\tau_{\vec{q}} \to \infty$ (the Peierls equilibrium problem) is revealed in the electrical conductivity expression. In general, the effects may be described as "phonon relaxation" effects, altering all the transport coefficients somewhat when $\tau_{\vec{q}}$ gets large, and "phonon-drag" or drift effects, altering appreciably only the thermoelectric power.

SPIN-LATTICE RELAXATION TIME FOR I^{127} IN KI IN THE TEMPERATURE RANGE 2°K TO 20°K. C. V. Briscoe* and C. F. Squire, The Rice Institute, Houston, Texas (Received August 4, 1958).

The spin-lattice relaxation time, T_1, for I^{127} in both single-crystal and powdered KI has been measured in the temperature range 2°K to 20°K using a Pound-Watkins-type nuclear magnetic resonance spectrometer. In the liquid hydrogen temperature range T_1 is approximately the same for both the single-crystal and the powder specimens, varying from 20 sec at 20°K to 70 sec at 14°K. Both the temperature dependence and the magnitude of T_1 in this temperature range can be explained on the basis of a theory of nuclear quadrupolar spin-lattice relaxation proposed by Van Kranendonk. In the liquid helium temperature range (2°K to 4°K), T_1 is constant within experimental error. For the single crystal $T_1 = 6.9 \times 10^3$ sec and for the powder specimen $T_1 = 6.0 \times 10^2$ sec in this temperature range. These

values of T_1 are less by orders of magnitude than is predicted using Van Kranendonk's theory. The I^{127} resonance (spin $I = 5/2$ in units of $\hbar$) consisted of a narrow (less than 1 gauss half width) central line with broad (>15 gauss) weak wings. This is attributed, according to Pound, to a shift in the nuclear magnetic energy levels brought about by the interaction of the nuclear quadrupole moment with electric field gradients present in the specimens.

*Now at the University of North Carolina, Chapel Hill, North Carolina.

THEORY OF SOLID Ne, A, Kr, AND Xe AT 0°K. Newton Bernardes, Department of Physics, Washington University, St. Louis, Missouri (Received April 10, 1958).

A quantum-mechanical variational technique is applied to an Einstein model of a solid, and the heats of sublimation and equations of state of solid Ne, A, Kr, and Xe are calculated at 0°K. Mie-Lennard-Jones 6-12 potentials appropriate to the gas phase data are used throughout, and the importance of quantum-mechanical effects is discussed; in general, good agreement with experiment is obtained. From the theoretical zero-point energies equivalent Debye temperatures, θ, are calculated, and from the dependence of these θ on volume, Grüneisen constants are computed in good agreement with experiment. Theoretical compressibility curves (at 0°K) are presented, and compared with the available experimental data; in the case of Ne, the only substance for which high-pressure data are available, the agreement is rather good up to 20 kiloatmospheres.

X-RAY DIFFRACTION AND MAGNETIC MEASUREMENTS OF THE Fe-Cr SPINELS. W. D. Derbyshire* and H. J. Yearian, Purdue University, Lafayette, Indiana (Received August 12, 1958).

X-ray diffraction intensity measurements and magnetic moment measurements were made on the spinel system $FeFe_{2-x}Cr_xO_4$, $0 \leq x \leq 2$, for the purpose of finding structural details in this system which were unknown. The x-ray measurements covered the entire composition range, while the magnetic measurements covered the range from $x = 0$ to $x = 1.2$. From the x-ray measurements the oxygen parameter of each sample was deduced. These parameters indicated inverse spinels for low values of x, partially inverse spinels for intermediate values of x, and normal spinels for $x = 1.3$ to $x = 2$.

The magnetic measurements indicated inverse spinels for $x = 0$ to $x = 0.3$. It was inferred from the magnetic measurements that the spins of cations on the B sites are not all parallel, at least for large x and probably for $x > 0.5$. Hence, the arrangement of cations on the A and B sites for a spinel in this composition range could not be deduced from its saturation magnetic moment. Estimates of the relative strengths of the magnetic interactions in various composition ranges and a prediction of the variation in saturation magnetic moment with composition in the range $x = 1.3$ to $x = 2$ were made.

*Present address: Physics Department, Colorado State University, Fort Collins, Colorado.

DECAY OF EXCESS CARRIERS IN SEMICONDUCTORS. K. C. Nomura and J. S. Blakemore, Honeywell Research Center, Hopkins, Minnesota (Received August 13, 1958).

A discussion is given of the nonlinear differential equations which govern the decay of excess carriers with arbitrary densities. The form of decay is explored for situations where the Fermi level is in the same half of the energy gap as the recombination level; criteria are established for both strong and weak trapping in addition to recombinative action. Analytic results are augmented and illustrated by numerically computed decay curves for a variety of circumstances. The separate solutions for holes and electrons are combined to show various kinds of behavior for photoconductive lifetime.

THEORY OF THE CONTRIBUTION OF EXCITONS TO THE COMPLEX DIELECTRIC CONSTANT OF CRYSTALS. J. J. Hopfield,* Physics Department, Cornell University, Ithaca, New York (Received July 16, 1958).

It is shown that the ordinary semiclassical theory of the absorption of light by exciton states is not completely satisfactory (in contrast to the case of absorption due to interband transitions). A more complete theory is developed. It is shown that excitons are approximate bosons, and, in interaction with the electromagnetic field, the

exciton field plays the role of the classical polarization field. The eigenstates of the system of crystal and radiation field are mixtures of photons and excitons. The ordinary one-quantum optical lifetime of an excitation is infinite. Absorption occurs only when "three-body" processes are introduced. The theory includes "local field" effects, leading to the Lorentz local field correction when it is applicable. A Smakula equation for the oscillator strength in terms of the integrated absorption constant is derived.

*Present Address: Bell Telephone Laboratories, Murray Hill, New Jersey.

NUCLEAR MAGNETIC RESONANCE IN CEROUS MAGNESIUM NITRATE AT TEMPERATURES BELOW 1°K. T. L. Estle, H. R. Hart, Jr., and J. C. Wheatley, Department of Physics, University of Illinois, Urbana, Illinois (Received June 2, 1958).

The nuclear resonance of the protons in cerous magnesium nitrate, $Ce_2Mg_3(NO_3)_{12}\cdot 24H_2O$, has been studied in a single crystal cooled to temperatures below 1°K by partial adiabatic demagnetization and adiabatic rotation. The area under the nuclear resonance was used as a thermometric parameter. The area did not obey Curie's law as a result of the temperature-dependent local rf field. A T-T^* relation was derived which gave temperature ratios on adiabatic rotation which agreed within 5-10% with ratios calculated from the properties of the crystal. The lowest measured temperature was 5×10^{-3}°K.

A study of the structure of the resonance showed that the splittings were caused by the average magnetization of the cerium ions and that the protons were at least 5A from the cerium ions.

ISOTOPE EFFECT ON THE SUPERCONDUCTING TRANSITION IN LEAD. R. R. Hake,* D. E. Mapother, and D. L. Decker, Physics Department, University of Illinois, Urbana, Illinois (Received April 10, 1958).

Observations of the difference in critical fields of superconducting Pb specimens of different isotopic mass are described. Measurements are made using the Meissner effect and a ballistic induction method in the range from 7.2 to about 1.4°K. The results near T_c verify the theoretical prediction that $T_c = \text{const} \times M^p$, where M is the average isotopic mass, and yield a value of $p=-0.501\pm0.013$. The measurements at lower temperatures are inconclusive in their implications for the isotope effect since all specimens measured exhibit large deviations from the predictions of the similarity principle. The deviations are apparently not related to the isotopic mass of the specimens. The agency responsible for the observed deviations has not been isolated, but it is believed that the anomalous H_c values are characteristic of the thermodynamically irreversible transitions. Thus the present results are not believed to constitute genuine evidence of a deviation from the similarity principle.

*Present address, Atomics International, Canoga Park, California.

MECHANISM OF THE FORMATION OF DONOR STATES IN HEAT-TREATED SILICON. W. Kaiser, H. L. Frisch, and H. Reiss, Bell Telephone Laboratories, Murray Hill, New Jersey (Received August 4, 1958).

A mechanism for the donor formation during heat treatment of silicon crystals is presented which accounts quantitatively for the complex kinetic phenomena and which is consistent with the known extra-kinetic information concerning this system. Atomically dissolved oxygen introduced during the growth of the silicon crystal reacts, in the course of heat treatment, to form a sequence of kinetically linked aggregates till finally a polymeric silica (SiO_2) is formed. Only those aggregates which possess fewer than five bound oxygen atoms appear to act intensively as donors at room temperature and in particular donor states produced around 450°C appear to consist principally of a donor [SiO_4] complex. The kinetic equations are integrated using a general purpose analog computer for a variety of initial concentrations of oxygen, temperature, etc., and the results compare favorably with existing experimental observations.

SOME ATOMIC EFFECTS OF AN ELECTRONIC ELECTRIC DIPOLE MOMENT. E. E. Salpeter, Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received August 4, 1958).

A parity-violating perturbation, corresponding physically to a permanent electric dipole moment

of an electron parallel to its spin, is introduced into the Dirac equation for an electron. For positrons two choices of the relative sign of electric moment and spin are considered. Some numerical consequences of such a perturbation for hydrogenic atoms and for positronium are calculated.

From various experiments carried out previously, rough upper limits are obtained for ξ, the electric dipole moment expressed in units of $e\hbar/mc$: From the numerical value of the Lamb shift, $\xi < 0.004$; from the metastability of the 2s state in hydrogen, $\xi < 0.03$. From the absence of K-L_{I} x-ray transitions in the heaviest atoms, the upper limit for ξ is about 0.005 or, probably, even smaller. From the value of the hyperfine splitting of the positronium ground state, $\xi < 0.02$.

MAGNETIC MOMENT OF HELIUM IN ITS $^3S_1$ METASTABLE STATE. C. W. Drake, V. W. Hughes, A. Lurio,* and J. A. White, Yale University, New Haven, Connecticut (Received July 30, 1958).

The ratio of the electronic g_J value of He in its $1s2s$, $^3S_1$ metastable state to the g_J of H in its ground state has been redetermined by the atomic beam magnetic resonance method using separated oscillating fields. At magnetic fields of about 540 and 575 gauss, the transitions $\Delta m = \pm 1, \pm 2$ in He and $(F, m) = (1,0) \leftrightarrow (1,-1)$ in H are observed at frequencies of approximately 1570 Mc/sec and 1140 Mc/sec, respectively. The natural line shapes for the separated oscillating fields method are obtained with spacings between the oscillating fields of both 4.2 cm and 2.2 cm. The experimental result is

$$g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})|_{\mathrm{exp}} = 1 - (23.3 \pm 0.8) \times 10^{-6}$$

which agrees with an earlier measurement having an accuracy of 16 ppm, and also with the theoretical value

$$g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})|_{\mathrm{theor}} = 1 - (23.3 \pm 1.0) \times 10^{-6}$$

obtained by Perl and Hughes, which was computed from the Breit equation for He with the addition of a term to represent the interaction of the anomalous spin magnetic moment of each electron with the external magnetic field.

*Now at Watson Laboratory, New York, New York.

TIGHT-BINDING CORRECTIONS OF STOPPING POWERS. Werner Brandt, Radiation Physics Laboratory, Engineering Department, E. I. du Pont de Nemours and Company, Incorporated, Wilmington, Delaware (Received August 7, 1958).

Theoretical tight-binding corrections for the stopping-power deficiency of inner-shell electrons at low particle velocities are compared with the experimental data measured by Burkig and MacKenzie for 20-Mev protons and by Bakker and Segrè for 300-Mev protons, with the following results. Walske's accurate tight-binding corrections for K and L shells are in complete agreement with experiment under conditions where only K and L shells are stopping-power deficient. If a larger fraction of the electron cloud of heavy atoms is deficient, extrapolations of Walske's corrections beyond their intended validity range underestimate the stopping-power deficiency. The stopping-power deficiency of heavy elements is well described by a cut-off approximation by Lindhard and Scharff of the tight-binding corrections of the statistical atom. "Best" tight-binding corrections are recommended for practical work, and areas for further theoretical work indicated. After correction for valence and tight-binding effects, the mean excitation potentials I_0 from both sets of experimental data are in mutual agreement. The decrease of I_0/Z with increasing Z confirms the $Z^{-2/3}$ dependence of the exchange effects in the statistical atom.

STEADY-STATE FREE PRECESSION IN NUCLEAR MAGNETIC RESONANCE. H. Y. Carr, Department of Physics, Rutgers University, New Brunswick, New Jersey (Received August 4, 1958).

A steady-state free precession technique for observing nuclear magnetic resonance is described. A mathematical analysis is presented for certain special conditions, and initial experiments verifying the results of this analysis are reported. This technique provides two opportunities for improving the signal-to-noise ratio. First, it provides a mechanism, similar to that of the "spin echo," for eliminating the effect of the inhomogeneity of the magnetic field on signal strength. This permits the effective use of larger samples. In the second place it provides a steady-state signal which can be ob-

served with a narrow-band detector. Under certain conditions the technique has a broad response as a function of frequency or field. The upper limit to the width of this response is determined by the electronic apparatus supplying the rf pulses rather than the magnet or the nuclear sample.

EVALUATION OF BREMSSTRAHLUNG CROSS SECTIONS AT THE HIGH-FREQUENCY LIMIT. U. Fano, H. W. Koch, and J. W. Motz, National Bureau of Standards, Washington, D. C. (Received August 4, 1958).

Recent experimental values for the bremsstrahlung cross section near the high-frequency limit with electron energies from 0.05 to 15 Mev and with target atomic numbers from 13 to 79 are compared with theoretical estimates. Calculations made to lowest order in $Z/137$ show large differences with the experimental results for large Z. Improved estimates, derived by detailed-balancing arguments from more accurate photoelectric cross sections, agree reasonably well with the experimental results throughout the indicated range of electron energies and atomic numbers.

LEVELS OF Cl^{33} FROM $S^{32}(p,p)S^{32}$ AND $S^{32}(p,p'\gamma)S^{32}$. J. W. Olness, W. Haeberli,* and H. W. Lewis, Duke University, Durham, North Carolina (Received June 23, 1958).

Differential cross sections for the elastic scattering of protons from S^{32} have been measured for the center-of-mass scattering angles 167.8°, 141.0°, 125.5°, 92.0°, and 63.9° for laboratory proton energies between 1.5 and 4.0 Mev. The cross sections for inelastic scattering to the 2.237-Mev level of S^{32} were also studied for incident proton energies between 2.7 and 3.8 Mev. These data were analyzed in terms of the single-level approximation of the dispersion theory. The results yield the following resonance energies (Mev) and assignments of spin and parity for the levels of the compound nucleus Cl^{33}: 1.900 $(3/2^-)$, 2.310 $(1/2^-)$, 2.578 $(5/2^-, 7/2^-)$, 2.810* $(3/2^-)$, 2.895 $(1/2^-)$, 2.902* $(5/2^-, 7/2^-)$, 2.917* $(3/2^+)$, 3.094* $(5/2^-)$, 3.195* $(5/2^+)$, 3.273 $(1/2^+)$, 3.379* $(7/2^-)$, 3.480 $(3/2^-)$, 3.570 $(1/2^+)$, and 3.716* $(5/2^-)$. Those resonances which exhibit observable widths for inelastic scattering are designated in the above tabulation by an asterisk. The analysis of the angular dependence of the inelastic cross sections supports the assignment 2^+ to the first excited state of S^{32}.

*Now at the University of Wisconsin, Madison, Wisconsin.

4+ EXCITED STATE IN OSMIUM 188. P. S. Fisher and R. A. Naumann, Palmer Physical and Frick Chemical Laboratories, Princeton University, Princeton, New Jersey (Received June 13, 1958).

Transitions in Os^{188} excited by the K capture from 41-hour Ir^{188} have been observed at 155, 324, 478, 633, 641, 827, 832, 1205, 1320, 1440, 1560, 1700, 1820, 1890, 1940, 2055, and 2190 kev. The new transition at 324 kev is in prompt coincidence with the 155-kev transition to the ground state, and an $E2$ assignment has been made from conversion-electron and gamma-ray intensities. It is proposed that this is the transition between a 4+ rotational level in Os^{188} at 479 kev and the 2+ state at 155 kev.

MEASUREMENTS ON THE LONGITUDINAL POLARIZATION OF BETA RAYS FROM P^{32}, Y^{90}, Pr^{144}, Au^{198}, AND Bi^{210} (RaE). J. S. Geiger, G. T. Ewan, R. L. Graham, and D. R. MacKenzie, Chalk River Laboratories, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (Received August 11, 1958).

The polarization of the beta rays from five negatron emitters has been studied by the method of Møller scattering in a magnetized NiFe foil. The scattered incident electron and the recoil electron from the target foil are detected in coincidence by two scintillation counters. The difference between the coincidence counting rates for the two directions of foil magnetization divided by the average rate gives a quantitative measure of the longitudinal polarization of the incident beta rays. Our experimental results are for P^{32}, $P=-(0.94\pm0.06)v/c$; Y^{90}, $P=-(0.86\pm0.06)v/c$; Pr^{144}, $P=-(0.90\pm0.22)v/c$; Au^{198}, $P=-(0.98\pm0.18)v/c$; and for RaE, $P=-(0.69\pm0.10)v/c$. The first four results may indicate a polarization magnitude slightly less than full $-v/c$. The differences are, however, within the range of possible systematic errors. The result obtained with RaE is significantly lower than the

average of the other four results. This indicates a departure from $-v/c$ polarization for this nuclide.

(p, α) REACTIONS INDUCED BY 23-Mev PROTONS. Clyde B. Fulmer and Bernard L. Cohen, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received July 1, 1958).

The (p, α) reactions induced by 23-Mev protons in targets of a wide range of atomic number were studied by observing the outgoing alpha particles. Alpha-particle energy distributions and absolute differential cross sections were measured at 30, 60, 90, 120, and 150 degrees. For targets of atomic number $\lesssim 50$, the energy spectra and angular distributions indicate that most of the alpha particles are produced in compound nucleus reactions; however, the Coulomb barriers of the excited compound nuclei are somewhat lower than those of ground-state nuclei. The alpha particles from the heaviest elements and the high-energy parts of the spectra from lighter targets are strongly forward peaked and are produced by direct interaction reactions. Total (p, α) cross sections vary from 175 mb for Al to 3.8 mb for Pt.

FIVE NEW ISOMERS WITH HALF-LIVES BETWEEN 10^{-6} AND 10^{-1} SECOND. Robert B. Duffield* and Stanley H. Vegors, Jr.,† Physics Department, University of Illinois, Urbana, Illinois (Received April 1, 1958).

Isomers with half-lives greater than approximately 5 μsec were sought between the 180-cycle, 1-μsec x-ray pulses of a 22-Mev betatron. The following new activities were observed:

Isotope	Half-life (μsec)	Gamma-Ray Energy (kev)	Production Reaction	Production Threshold (Mev)
$Br^{78,80}$	127 ± 5	149 ± 6	γ, n	11.0 ± 0.3
$Br^{79,81}$	37 ± 3	268 ± 4	γ, γ'	. . .
Nb^{92}	5.9 ± 0.5	88 ± 5	γ, n	9.1 ± 0.7
In^{114}	(42 ± 5)×10^3	311 ± 5	γ, n	9.9 ± 0.2
Sn	165 ± 15	117 ± 5	γ, n	~11
		163 ± 5		
		504 ± 6		

Excitation functions and cross sections for the production of the above isomers are also given for some cases. The properties of all the isomers investigated are summarized in this paper. Additional results include improved values for half-lives and gamma-ray energies, plus some measurements on production thresholds, excitation functions, and cross sections for the isomers listed in a previous paper.

*Present address: John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, P. O. Box 608, San Diego 12, California.

† Present address: Atomic Energy Division, Phillips Petroleum Company, Idaho Falls, Idaho.

EXCITED STATES OF V^{51} AND Cr^{53}. M. Mazari,* W. W. Buechner, and A. Sperduto, Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received August 4, 1958).

The region of excitation in V^{51} up to 4.12 Mev has been investigated through studies of the inelastic scattering of protons. In this region of excitation, thirty-five excited states were found. The low-lying excited states of Cr^{53} were investigated through the $Cr^{52}(d, p)Cr^{53}$ and the $Mn^{55}(d, \alpha)Cr^{53}$ reactions. The ground-state Q values for these reactions are 5.720 ± 0.006 and 8.275 ± 0.008 Mev, respectively. The first two excited states in Cr^{53} are at 0.565 and 1.008 Mev. In these studies, the bombarding beam was provided by an electrostatic accelerator, and the reaction products were analyzed with a broad-range magnetic spectrograph.

*Now at the National University of Mexico.

HIGH-RESOLUTION ELECTRON-SPECTROSCOPIC STUDY OF THE 60-HOUR ELECTRON-CAPTURING ISOMER Re^{182}. C. J. Gallagher, Jr., and J. O. Rasmussen, Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received April 14, 1958).

A thorough study of the conversion electron spectrum from electron capture decay of the 60-hr Re^{182} isomer is reported. A very large number of transitions are observed, including most of those reported by previous investigations on the beta decay of Ta^{182} to the same daughter nucleus W^{182}. In addition many new transitions are observed. Relative intensity comparisons from the present study add weight to a Ta^{182} de-

cay scheme published by earlier investigators. Using the Ta^{182} decay scheme as a base, it was possible to fit most of the new transitions into a plausible decay scheme involving several new levels. A tentative grouping of many of the new levels into rotational bands is proposed, mainly on the basis of relative intensities of gamma transitions depopulating various levels.

*Present address: Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California.

NITROGEN-NITROGEN ELASTIC SCATTERING. Charles E. Porter, University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (Received July 24, 1958).

An optical-model analysis of the measurements of nitrogen-nitrogen elastic scattering is made, and a quasi-classical discussion of the corresponding phase shifts is given.

COULOMB EXCHANGE ENERGY FROM SHELL MODEL WAVE FUNCTIONS. N. V. V. J. Swamy, The Florida State University, Tallahassee, Florida, and A. E. S. Green,* Los Alamos Scientific Laboratory, Los Alamos, New Mexico (Received June 13, 1958).

The direct and exchange Coulomb energies of several light nuclei have been calculated by using harmonic oscillator wave functions and approximate wave functions for a realistic diffuse boundary potential. The Z dependence of the total exchange energy is expressed in terms of a convenient parameter c. Calculations show that for very low Z this parameter is appreciably smaller than a prior estimate by Bethe and Bacher based upon the statistical model but that it rapidly reaches their value $5(3/16\pi)^{2/3}$ for $Z \geq 10$.

*On leave from Florida State University until September, 1958.

MEASUREMENTS OF THE INTERACTION OF 95-Mev PROTONS WITH He^4. W. Selove* and J. M. Teem,† Harvard University, Cambridge, Massachusetts (Received May 26, 1958).

Elastic and inelastic scattering and the (p, d) "deuteron pick-up" process have been studied using a proton beam of energy about 95 Mev incident on a liquid helium target. The elastic scattering shows a nuclear-Coulomb interference dip at 9°, a slight diffraction type minimum at about 57°, and a deep minimum, approximately 10^{-29} cm^2/sterad, at 135°. (All angles and cross sections are in the center-of-mass system.) Inelastic scattering spectra were obtained at laboratory angles of 10°, 15°, and 30°. These spectra are characterized by broad peaks, roughly 10 Mev wide, centered around an energy about 6 Mev below the upper kinematical limit for inelastic scattering. Their interpretation is discussed qualitatively both in terms of quasi-elastic nucleon-nucleon scattering and in terms of strong interaction between parts of the dissociated α particle in virtual or continuum states. A minimum was observed in the $He^4(p, d)He^3$ differential cross section at about 29°. Analysis of the (p, d) data at 95 Mev and 32 Mev in the "transparent nucleus" Born approximation yielded inconsistent results; presumably this inconsistency is due to the failure of the model at the lower energy due the tightly bound structure of He^4.

*Now at the University of Pennsylvania, Philadelphia, Pennsylvania.

†Now at the California Institute of Technology, Pasadena, California.

PROTON ANGULAR DISTRIBUTIONS FROM $Zn^{64, 66, 67, 68}(d, p)$ REACTIONS. F. B. Shull and A. J. Elwyn,* Wayman Crow Laboratory of Physics, Washington University, St. Louis, Missouri (Received July 28, 1958).

Angular distributions and Q values were determined for proton groups from (d, p) reactions with isotopically enriched targets of Zn^{64}, Zn^{66}, Zn^{67}, and Zn^{68}. The incident deuteron energy was 10 Mev. The angular distributions were fitted by Butler curves, from which values of l_n were determined, where $l_n\hbar$ is the orbital angular momentum of the captured neutron. The fits were only fair, presumably because the Butler theory cannot be expected to give good fits at $Z = 30$ without modification. In addition, several peaks are evidently superpositions of closely spaced, unresolved groups with different l_n. Well resolved empirical distributions which appear to correspond to $l_n = 0$, 1, 2, 3, and 4 were observed, which serve to indicate in which direction the Butler curves should be shifted, thus increasing confidence in the correctness of the values of l_n which were assigned. For $Zn^{64}(d, p)$, we find $Q = 5.71$ Mev with $l_n = 1$ and 3, Q

= 4.89 with $l_n = 1$ and 3 or 4, $Q = 4.43$ Mev with $l_n = 2$, $Q = 3.86$ Mev with $l_n = 0$, and $Q = 3.31$ Mev with l_n = perhaps 2 and 4. For $Zn^{66}(d,p)$, the results were $Q = 4.76$ Mev with $l_n = 1$ and 3, $Q = 4.38$ Mev with $l_n = 1$ and 3 or 4, $Q = 3.85$ Mev with $l_n = 2$, and $Q = 3.15$ Mev with l_n uncertain. For $Zn^{67}(d,p)$ we find $Q = 8.01$ Mev with $l_n = 3$, $Q = 6.90$ Mev with $l_n = 1$, $Q = 6.13$ Mev with l_n unknown, and $Q = 4.52$ Mev with l_n = perhaps 1 and 3. For $Zn^{68}(d,p)$, the results were $Q = 4.22$ Mev with $l_n = 1$, $Q = 3.78$ Mev with $l_n = 4$, and $Q = 3.40$ Mev with $l_n = 2$.

*Presently at Brookhaven National Laboratory, Upton, New York.

ELASTIC SCATTERING OF 21.6-MEV DEUTERONS. J. L. Yntema, Argonne National Laboratory, Lemont, Illinois (Received June 19, 1958).

The absolute differential cross section for the elastic scattering of (21.6 ± 0.2)-Mev deuterons has been measured for Mg, Ni, Cu, Zn, Rh, Ag, Pt, and Au. The cross sections were measured for each element with a two-degree interval between 10° and 42° and between 45° and 150° with a five-degree (or smaller) interval. The estimated standard deviation ranges from 3% in the forward direction to 6% in the backward direction. The deuterons were detected with a double NaI(Tl) crystal with an energy resolution of about 2%. In most cases the contribution of inelastically scattered deuterons to the cross section is negligible. The ratio of measured cross section to Rutherford cross section shows a pronounced diffraction pattern in all cases.

NEUTRAL DECAY MODES OF THE θ_1^0 AND Λ^0. Elihu Boldt,* Herbert S. Bridge, David O. Caldwell, and Yash Pal,† Massachusetts Institute of Technology, Cambridge, Massachusetts, and Brookhaven National Laboratory, Upton, New York (Received July 30, 1958).

In a large multiplate cloud chamber, an event has been found which strongly indicates that the decay mode $\theta_1^0 \to 2\pi^0$ exists, and therefore that the spin of the K meson is even. Two showers are seen which, from their spatial angles and energies, must represent essentially the whole energy of the two π^0's from a θ_1^0 produced in association with a Λ^0. On the basis of this event, the fraction of θ_1^0 decays going by the $2\pi^0$ mode is ~0.06, and the probability that this fraction could be consistent with the value of 1/3 predicted by $\Delta I = 1/2$ selection rule is 3%. Furthermore, the chance that the observed ratio is $\geq 1/4$, a value possible with an admixture of $\Delta I = 3/2$, is 10%. Our observation of single shower events attributable to $\theta_1^0 \to 2\pi^0$ give even smaller probabilities that the fraction of neutral decays is as large as 1/3 or 1/4, but the necessity of a background subtraction makes these results less significant. On the other hand, the fraction of Λ^0 decays going by a neutral decay mode is quite consistent with the value of 1/3 predicted by the $\Delta I = 1/2$ rule. The values obtained for this fraction are 0.22 ± 0.13 from the direct observation of showers (presumably from $\Lambda^0 \to n + \pi^0$), and 0.33 ± 0.06 from the number of Λ^0's not decaying by the charged mode.

*Present address: Department of Physics, Rutgers University, New Brunswick, New Jersey.

† Now at the Tata Institute of Fundamental Research, Bombay, India.

SCATTERING OF 80-Mev π^- MESONS BY COMPLEX NUCLEI. W. F. Baker and J. Rainwater, Columbia University, New York, New York, and R. E. Williams, Paul Rosenberg Associates, Mount Vernon, New York (Received August 4, 1958).

The elastic scattering of 80-Mev π^- mesons by lithium, carbon, aluminum, and copper has been investigated using a π^- detector which gives improved discrimination against low-energy-loss inelastic events. A "transmission" scatterer orientation was necessary to maintain energy resolution, so only less than 120° scattering was studied. The new differential scattering cross sections for lithium, aluminum, and copper agree with the previously reported results at small angles but are 2 to 10 times smaller at larger angles. There are indications of diffraction structure in aluminum and copper which were not previously observed due to the poorer energy resolution of the earlier measurements.

Including the incident beam energy spread, the detector sensitivity is down by factors of 3 and 9 for $E \pm 5.0$ Mev and $E \pm 10$ Mev when peaked for mean energy E. This was used to separate the scattering from carbon into elastic and inelastic components for excitation of the 4.43- and (7.65- and 9.61-) Mev levels. Inelastic scatterings from lithium have also been studied. For the

same momentum transfer on scattering, the ratio of the inelastic scattering with excitation of the 4.43-Mev and 9.61-Mev levels to elastic differential cross section is the same, within experimental accuracy, as found by Fregeau for the scattering of 187-Mev electrons on carbon. The elastic scattering is compared, in the following Article, with the results of complex square-well phase shift calculations, and with diffuse edge optical model and modified Kisslinger model calculations.

THEORECTICAL CALCULATIONS ON THE SCATTERING OF π^- MESONS BY COMPLEX NUCLEI. W. F. Baker, H. Byfield, and J. Rainwater, Columbia University, New York, New York (Received August 4, 1958).

The angular distributions of 80-Mev negative pions elastically scattered from Li, C, and Al have been analyzed on the basis of optical models using IBM 650 computers. It was not possible to fit the data with the usual $V_1 + iV_2$ type model having either a square or diffuse edge and V_1 and iV_2 proportional to the nuclear density. A Kisslinger type model, which takes better account of the p-wave nature of the basic π-nucleon scattering process, was also used with a modification to avoid a physically unreasonable behavior of the effective interaction. Using $r_0 = 1.08$ and a diffuseness parameter $a = 0.25$ in a Saxon type density distribution, good agreement with experimental data was obtained.

EXAMPLE OF THE PRODUCTION OF A π^- MESON BY A K^+ MESON. E. Helmy, J. H. Mulvey,* D. J. Prowse, and D. H. Stork, Department of Physics, University of California, Los Angeles, California (Received May 21, 1958).

In 50 meters of 200- to 300-Mev K^+-meson track followed in nuclear emulsion, one interaction event has been found from which a pion track emerges. The incoming K^+ meson had a kinetic energy of 323 ± 20 Mev. The K^+ meson re-emerged with an energy of 30 Mev at 36 degrees. The pion track has a characteristic negative-pion ending, and the pion was emitted with 34 Mev at 96 degrees. In addition a 40-Mev proton left the star at 81 degrees and there are three other short tracks. Although the kinematics admit an analysis in which no inelastic secondary process occurred among the primary reaction products, strong inelastic secondary processes are quite possible and must be considered. Charge independence holds important implications for the pion yield charge ratios and the early trend of negative excess seems rather striking.

*Now at the Clarendon Laboratory, Oxford University, England.

DECAY MODES OF A $(\theta + \bar{\theta})$ SYSTEM. M. Goldhaber, T. D. Lee,* and C. N. Yang,† Brookhaven National Laboratory, Upton, New York (Received August 12, 1958).

Relations among the different states of a system consisting of a θ meson, a $\bar{\theta}$ meson, and a certain number of pions are discussed. Theorems concerning certain correlations between the production probabilities for charged and uncharged θ mesons and the decay modes (θ_1^0 and θ_2^0) are obtained.

*Permanent address: Columbia University, New York, New York.

†Permanent address: Institute for Advanced Study, Princeton, New Jersey.

ACCELERATION OF COSMIC RADIATION. G. S. Murty, and R. K. Varma, Tata Institute of Fundamental Research, Apollo Pier Road, Bombay, India (Received August 8, 1958).

Alfvén's model of discontinuous magnetic field variations for acceleration of cosmic rays by the betatron mechanism has been generalized to continuous field variations, with special reference to a sinusoidal field. It is shown that the efficiency of this model is maximum when the "rate of randomization" is equal to the frequency of oscillation of the magnetic field. Also the maximum efficiency is comparable with those of Alfvén's and Parker's models.

DISCREPANCY BETWEEN π^--PROTON SCATTERING AND A DISPERSION EQUATION. Howard J. Schnitzer and George Salzman, Department of Physics and Astronomy, University of Rochester, Rochester, New York (Received August 7, 1958).

The π^--proton scattering is reanalyzed following Puppi and Stanghellini. The use of more recent data in evaluating the very sensitive princi-

pal-value integral and a more detailed error analysis lead, with the same coupling constant, $f^2=0.08$, and the same S-wave scattering lengths, $a_1=0.165$ and $a_3=-0.105$, to much less disagreement than they found. However, a residual discrepancy of the same general character remains. It is pointed out that the analysis is very sensitive to the experimental charge-exchange scattering, and two alternatives to using this data are suggested. The importance of D waves, in extending the analysis to energies above 300 Mev, is demonstrated. It is shown that contributions from energies above ~ 2 Bev, and reasonable changes in f^2, a_1, and a_3, cannot contribute significantly towards resolution of the discrepancy.

SUPRATHERMAL PARTICLE PRODUCTION BY BINARY STARS. D. A. Tidman, The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois (Received July 21, 1958).

It is suggested that the expanding coronas of binary stars collide and produce a region in violent motion capable of accelerating ions by the Fermi mechanism. This may be important for injecting low energy cosmic radiation into the interstellar medium.

VARIATIONS IN THE COSMIC-RAY NUCLEONIC INTENSITY. J. A. Lockwood, University of New Hampshire, Durham, New Hampshire (Received June 13, 1958).

The monthly mean intensity of the nucleonic component of cosmic radiation measured at Mt. Washington, geographic latitude 44.2°N and elevation 6262 ft, decreased 22% from July, 1954, to December, 1957. The decreased intensity now prevailing came about through a series of sudden decreases from which only partial recovery occurred. During this same period, the amplitude of the day-to-day intensity changes has increased. A study has been made of the largest Forbush-type decreases which have occurred in this interval. Many of these intensity drops have a decrease rate ~1.5% hr^{-1}, with a maximum rate of 2.5% hr^{-1}. In some cases they are preceded by a slight increase of ~1.5%. These decreases do not have a similar time dependence. There is no apparent 27-day recurrent tendency for these sudden drops. The energy dependence of these events is variable. The restrictions which these decreases impose upon the existing models of solar modulation mechanisms are discussed.

EFFECT OF A COMPLEX SPIN-ORBIT COUPLING ON THE POLARIZATION OF PROTONS ELASTICALLY SCATTERED FROM NUCLEI. R. M. Sternheimer, Brookhaven National Laboratory, Upton, New York (Received August 15, 1958).

The effect of a complex spin-orbit coupling on the polarization in elastic nucleon-nucleus scattering has been investigated for the particular case of 150-Mev protons scattered from emulsion nuclei (Ag, Br, average $Z=41$). The calculations were carried out for a fixed central potential V and a fixed real part U_r of the spin-orbit coupling, and with a variable imaginary part U_i of the spin-orbit coupling. It was found that the maximum polarization P_{max} inside the first diffraction peak increases somewhat with increasing U_i (from 0.55 for $U_i=0$ to 0.76 for U_i with maximum value $U_{i,\,max}=1.0$ Mev on the surface of the nucleus). The small negative values of the polarization P in the small-angle region of Coulomb scattering ($P\sim-0.02$) and the large negative values of P in the region of the first diffraction minimum are rather insensitive to changes of U_i.

UNUSUAL COSMIC-RAY INTENSITY FLUCTUATIONS OBSERVED AT SOUTHERN STATIONS DURING OCTOBER 21-24, 1957. K.G. McCracken, Physics Department, University of Tasmania, Hobart, Tasmania, and N. R. Parsons,* Antarctic Division, Department of External Affairs, Melbourne, Australia (Received December 17, 1957; revised manuscript received May 19, 1958).

Observations are reported from four southern stations of cosmic-ray intensity fluctuations during the period October 21-24, 1957. These fluctuations exhibit unusual features which suggest the existence of a short-lived and highly directional anisotropy of the primary radiation during the period immediately preceding a Forbush-type decrease.

*Present address, Physics Department, University of Tasmania, Hobart, Tasmania.